A Binomial Model with Edgeworth Expansion on Particular Circumstances

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ABSTRACT

Contexts with high volatility and extreme events condition the value of the firm, its tax savings and continuity. These conditions must be contemplated for the employed valuation model. In that sense, the present paper' basis is the classic binomial model incorporating: a) firm contingent states of continuity or dissolution; b) tax saving valuation like a basket of real options, and c) extreme events by Edgeworth transformation. The paper structures in the following manner: first it develops the binomial function changed with the Edgeworth extension and the construction of implicit binomial lattice. Then it develops a valuation adapted to the binomial model with Edgeworth expansion that incorporates contingent tax savings, continuity or liquidation scenarios and cost of bankruptcy. With a hypothetical case it is illustrated its functioning, and comparing the results obtained between situations with kurtosis and skewness or normally. Finally the main conclusions are exposed.

KEYWORDS

1. Introduction

Nowadays, firms are living in high volatility contexts and exposed to small probability events but with high impact on decisions and company's worth value. As a consequence, projecting variables such as results, worth value, or financial costs assuming normal distribution might be a false estimation of what will occur in reality in businesses resulting on a poor valuation process. Thus, we must develop models where random variable estimations include high-order moments. In other words, we must introduce asymmetry and kurtosis in order to capture biases and fat-tailed distribution related to extreme events. A well-known method used in firm valuation is the discounted cash flow. Depending on how they treat cost of capital and tax savings, we could classify them into: a) Weighted Average Cost of Capital (WACC), Capital Cash Flow (CCF), and Adjusted Present Value (APV) (Ruback, 2002; Booth, 2002; Damodaran, 2006; Booth 2007; Fernandez, 2014). On its traditional version, discounted cash flow methods present controversial issues related to valuing tax savings as a result of using financial debt and its impact on firm and equity worth value because of futures scenarios conditioned to liquidation as a result of financial difficulties. These models assume firm's value as a lineal relationship generated by positive tax savings and projecting expected average cash flows that sum up potential scenarios. This simplifying assumption has nothing to do with real business because of the following reasons: a) Tax savings: their existence is conditioned by positive results, the operative earnings tax being equal or higher than the tax savings; b) firm and equity worth value: these models must include contingent scenarios of continuity and liquidation with free cash flows higher than debt flows (firm's continuity) or insufficient free cash flows (automatic firm's liquidation); c) Including extreme events and biases: this could be achieved modifying the probability distribution function of random variables (results, firm worth value, and cost of debt) with the Edgeworth expansion that introduces asymmetry and kurtosis.

The objective of this paper is to develop a valuation model that considers: a) contingent value of tax savings; b) continuity scenarios and firm liquidity; c) extreme events that transforms probability distribution. In order to achieve our first objective, we propose to value tax savings as they were the cash flow of a financial option (Velez Pareja, 2016). Regarding our second objective, binomials models will be adapted from previous works (Broadie & Kaya, 2007; Milanesi, 2014). These papers as based on the classical concept of considering shareholders' equity as a call option, but in contrast to these publications

and to presenting an original work consistent with the tax savings treatment, we assume that firm value follows a geometric Brownian motion (Brandao, Dyer & Hahn, 2005; Smith, 2005). In addition, operative earnings before taxes are described by an arithmetic Brownian motion. Finally, our third objective is achieved incorporating the Edgeworth expansion into the binomial valuation model (Rubinstein 1998; Milanesi, 2013).

This paper will be structured in the following manner: in the next section, we will develop a series of equations in order to explain the Edgeworth transformation on the binomial model, implicit probabilities, tax savings value as in an option portfolio, and the binomial valuation model conditioned to liquidation results. Subsequently, we will present a practical example assuming normal behavior from random variables (symmetry and mesokurtic) and a negative and extreme biased (asymmetric and platykurtic). We will contrast the obtained results between the proposed model and the traditional discounted cash flow model. Finally, we will present our main conclusions.

2. Edgeworth Expansion

2.1. Edgeworth Expansion in Binomial Distribution and Implicit Probabilities

This model uses a binomial probability distribution $[b(x)]$ in order to project the behavior of: earnings(E), firm value(V), financial debt yield(i). In general, random variable (x) is present in its path $n+1$, final nodes, and $j=0, 1, 2,…n$ positions. The number of potential different paths is determined by,

$$r_j = \frac{n!}{j!(n-j)!} \ (1)$$

The value for every position is,

$$[(2j) - n]/\sqrt{n} \ (2)$$

Binomial probability function *b(x)* for every node is,

$$\left[\frac{n!}{j!(n-j)!} \ q^j \times (1 - q)^j\right] (3)$$

In order to incorporate asymmetry and kurtosis to the stochastic process in the binomial method, it is necessary to transform function *b(x)* (equation 3). On the binomial function *b(x)*, values related to four moments (mean, variance, asymmetry, and kurtosis) are *E(x) = 0; E(x²)= 1; E(x³) = 0, E (x⁴)=3*. Assuming a value different from 0 and 3 to higher moments, means getting away from normality and requires to apply the transformation

on the original function. Jarrow and Rudd (1982) apply the Edgeworth expansion to the binomial model from a technique developed by Schleher (1977) where the real probability distribution *f(x)* is now approached by a different one named *w(x)*. Through statistic distribution, this technique is known as the Edgeworth expansion (Cramer, 1946, Kendall & Stuart, 1977). This expansion approached a probability distribution that is more complex as could be the normal or lognormal distribution. This technique enables coefficients to be moments not only for the original distribution, but also for the approached one. The result is a new function *g(x,)* where the following moments are captured: $E(x) = 0$; $E(x^2) = 1$; $E(x^3) = E$, $E(x^4) = K$ (Rubinstein, 1998) from the following five steps:

a) We must calculate the transformation function with the following expression[1];

$$w(x) = [1 + 1/6\,E(x^3 - 3x) + 1/24(K - 3)(x^4 - 6x^2 + 3) + 1/72E^2(x^6 - 15x^4 + 45x^2 - 15)] \ (4)$$

The transformed function is the product between equations 3 and 4 on every node, so *g(x)* = *b(x)w(x)*. The expansion is only an approximation being $\sum_j g(x_j) \neq 1$. Probabilities must be weighted so they add 1, replacing $g(x_j)$ with $f(x_j)/\sum_j g(x_j)$.

b) Once we have got the adjusted density function, we proceed to estimate mean *m* and its variance (v^2);

$$\mu \equiv \sum_j g(x_j)x_j \ (5)$$

$$v^2 \equiv \sum_j g(x_j)(x_j - m)^2 \ (6)$$

Having equations 5 and 6, the necessary parameters come up in order to standardize random variables (projected results, interest yield, or firm value).

c) Higher moments are incorporated on mean and variance. Transformation function *w(x)* is applied on the binomial function *b(x)*, originating the transformed function *g(x)*. At the same time, random variables x_j are replaced by the now standardize with the following expression,

$$x^{g(x)} = (x_j - m)/v \ (7)$$

Having the new function *g(x)* and the inclusion of higher moments on mean and variance, we proceed to project the underlying asset value.

d) The random variable value on every node is denoted as V_j . It is calculated using the corrected function *g(x)*. Inputs are: growth rate (μ); obtained probabilities from the

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[1] Having asymmetry E=0 and kurtosis K=3, the transformation gets canceled, and the function gets back to the binomial normal estate.

corrected function *g(x)* are denoted as $p_j = g(x_j)$, associated to the underlying value on the option strike date, and standard deviation σ,

$$V^E_j = V_0 e^{\mu \times t + \sigma \sqrt{t} \times x_j} \ (8)$$

Before its estimation, we must operate on equation 9 to get the growth rate expression (μ);

$$(r/d)^t = \sum_j p_j (V_j/V) \ (9)$$

Variables involved in the equation are *V*= project intrinsic value at the initial moment; *r*= risk free rate; *d*= asset return yield; *t*= time until the decision must be made; *μ*=expected risk free increment from the logarithm of V_j/V and *σ*= volatility of the logarithm of V_j/V. Once we replace equation 8 on equation 9, applying logarithms and clearing from the risk free increment (μ) (equations 10 and 11),

$$(r/d)^t = \sum_j p_j \, e^{\mu \times t + \sigma \sqrt{t} \times x_j} = \left(\sum_j p_j \, e^{\sigma \sqrt{t} \times x_j} \right) e^{\mu \times t} \ (10)$$

$$\log[(r/d)^t] = \log \left(\sum_j p_j \, e^{\sigma \sqrt{t} \times x_j} \right) + \mu \times t \, (11)$$

We find the following equation to project the growth rate incorporating higher moments, necessary input of equation 8,

$$\mu = \log[(r/d)] - \frac{\left[\log \left(\sum_j p_j \, e^{\sigma \sqrt{t} \times x_j)} \right) \right]}{t} \ (12)$$

Growth rate *μ* is similar, theoretically, to the one we use to estimate risk neutral values assuming a lognormal distribution, *μ=[log(r/d)-1/2σ²]* (Rubinstein, 1998). Once we incorporate higher moments, we abandon the lognormal distribution assumption. Having these elements, we are ready to project value for different nodes. The following equation, summarize present value according to the successive projected values,

$$V^E_0 = \sum_j p_j (V^E_j) \times \rho^{-t} \ (13)$$

On this case, V^E_j is estimated with equation 8, *pj=g(xj)* with equations 4 and 7, and *ρ* is the risk free discounted rate.

e) We must estimate implicit probabilities for every node through backward induction (Rubinstein, 1998).

Process starts at terminal nodes, estimating probabilities with the following expression.

$$q = q'_j / \frac{n!}{j!(n-j)!} \ (14)$$

Once we define V^E as the underlying asset value, the two subsequent nodes are ($q_t^+ \, V_t^{+e}$; $q_t^- \, V_t^{-e}$). These are probabilities conditioned by the precedent node (q_{t-1}; V_{t-1}). The probability associated to node (q_{t-1}) is equal to the sum of the following subsequent nodes

$q_{t-1} = q_t^+ + q_t^-$. The precedent node summarizes the underlying's movements and probabilities (q_t^+ V_t^{+e}; q_t^- V_t^{-e}).On every node, certain equivalents are obtained by applying conditional probability. Equations are,

$$p_{jE(t)} = {q_t^+}/{q_{t-1}} \ (15)$$

$$1 - p_{jE(t)} = {q_t^-}/{q_{t-1}} \ (16)$$

V_{t-1} comes up from the product between subsequent brunches (V_t^{+E} ; V_t^{-E}) and certain equivalents coefficients (p_{jEt}; 1-p_{jEt}), discounting at a risk free rate related to time interval. Consequently, we apply backward induction to value the underlying asset and the option within itself (equation 17).

$$V_{(j-1,t-1)}^E = \left[p_{jE(t)} \times V_{(j-1,t-1)}^{E,+} + (1 - p_{jE(t)}) \times V_{(j-1,t-1)}^{E,-} \right] \ (17)$$

2.2. Tax Savings Present Value as an Option Portfolio and the Edgeworth Expansion

Traditionally, tax savings value for a period is estimated by,

$$V_T^{AF} = IF \times \tau \ (18)$$

where *IF* represents computable financial interest magnitude and τ is the tax marginal rate. Successive tax savings present value is equal to

$$VA(V_T^{AF}) = \frac{IF \times \tau}{r} \ (19)$$

This previous equation presents a debate between academics and practitioners. On this particular case, r is the discounted rate which is used on tax savings cash flows. There exists a discussion which discounted rate should be used. There are two extreme positions: a) Modigliani and Miller (1963) propose discounting tax savings at the risk free rate; b) Miles and Ezzell (1980, 1985) propose discounting tax savings at the debt yield during the first year, and discounting at the cost of capital of an unleveraged firm k_u for the following years. There are also some middle ground positions (Taggart, 1991; Inselbag & Kaufold, 1997; Tham & Wonder, 2001; Tham & Velez Pareja, 2001; Tham & Wonder, 2002; Booth, 2002; Farber, Gillet & Szafarz, 2006; Cooper & Nyborg, 2006; Oded & Michel, 2007; Velez Pareja, 2016). It is held that tax savings are conditioned by the capital structure objective whether by maintaining a fixed debt present value (Modigliani & Miller, 1963) or by keeping a fixed debt/equity market ratio (Miles &

Ezzell, 1980, 1985). Fernandez (2014) discards these positions [2]and, supported by papers where he proves that debt to equity ratio stays constant, proposes to estimate a debt to equity ratio based on book values. This occurs, partially, because administrators strongly consider book value since rating agencies keep an eye on them constantly (Flannery & Rangan, 2006). Accordingly, there is a debate about it. Copeland, Koller and Murrin (2000), declare that financial literature does not provide a clear answer related to what discount rate for tax savings in the correct one.

Applying option theory, this debate is solved. Tax savings contingent value is conditioned to the existence of positive results. This occurs since in options, risk is treated on cash flows, and risk free rate is simply used to reflect the time value of money.

When we do not expect changes on tax legislation, the only source of risk is determined by the variability of firm results. Thus, tax savings are subject to: a) the existence of positive results, b) results are equal or higher to the tax savings. If not, its deduction operates until the imputable operative earning value.

$$AF = \begin{cases} \text{EBIT} < 0; (0) \\ 0 < \text{EBIT} + \text{OI} < IF; (\text{EBIT}) \times \tau \ (20) \\ \text{EBIT} \geq \text{IF}; (\text{IF}) \times \tau \end{cases}$$

We must apply equations 18 and 19 if we could verify conditions a) and b). However, if we want to incorporate this third situation, we must apply real option theory. Tax savings is similar to a portfolio where we have a long position and a short position on an American call options. In other words, caps strategy being the underlying asset, the tax imputable base, composed by the operative results $S_t = EBIT_t$. Tax savings is equal to the algebraic sum between a long positiong on an American call $C(0)_t$, with a strike price $X = 0$, and a short position on an American call $C(IF)_t$, with a strike price equal to the imputable tax saving $X = IF \times \tau$.

There are three flows generated by the option portfolio similar to the tax savings. Flow 1: No exercise, inexistence of tax savings: $EBIT < 0; (0)$. Flow 2: Exercising the option. Option value will depend on the operative earnings: $C(0)_T = \max(EBIT \times \tau; 0)$. Exercising the long position will go from $0 < EBIT < IF; (EBIT) \times \tau$ to infinity. Flow 3: Selling the call option (tax savings) $C(IF)_T = \min\big((EBIT) \times \tau; (IF) \times \tau\big)$. Exercising the short position will go from $EBIT + OI \geq IF; (IF) \times \tau$ with its cap.

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2 Modigliani and Miller consider that debt present value stay constant, and Miles and Ezzell assume debt value as a multiple of equity market value.

Cash flows from the resulting portfolio come from adding the premium earned by the short position as a result of the tax saving (flow 3) and the cash flow that we lose by not exercising the long position (flow 2). In other words, portfolio value comes from the difference between the projected operative earnings tax and the projected earnings tax, conditioned to cash flows and precedent exercises $C(AF)_T = C(0)_T - C(IF)_T$. The synthetic expression relays to the terminal value related to the option portfolio,

$$C(AF)^E{}_T = \min\{[\max(EBIT^E) \times \tau; \ 0]; \ (IF^E) \times \tau\} \ (21)$$

On this case, all the expressions have with the *e* index that indicates a transformed variable incorporating higher moments through the Edgeworth expansion. Therefore, the underlying (EBIT) is assumed to incorporate asymmetry and kurtosis as the debt interest rate (i), so $IF^E = i^E \times P$ being P firm debt adapting equation 8. Once we project stochastic variables $EBIT^E$ and IF^E on every node $t = 1 \dots T$ and node (i,j) on the binomial tree, we must estimate the tax savings present value,

$$V_{T-t}^{AF}{}^E = C(0)_{T-t} - C(IF^E)_{T-t} + V_{T-t+1}^{AF}{}^E e^{-rf} \ (22)$$

Term $V_{T-t+1}^{AF}{}^E e^{-rf}$ represents tax savings for period $T - t + 1$ discounted at the risk free rate. For every node on moment T, we assume continuity of tax savings estimated on equation 21. From node *T-t*, tax savings is integrated between call option $C(AF)_T{}^E$ and expected tax savings present value on *T-t+1* for *(i,j)* $V_{T-t+1(i,j)}^{AF}{}^E e^{-rf}$.

Applying obtained probabilities for the transformed variable according to equations 15 and 16 on nodes *(i,j)*, we determine tax savings thinks as an American cap and floor equal to $IF^E \times \tau$,

$$V_{T-t(i,j)}^{AF}{}^E = p_{jE(t)} \times (V_{T-t+1(i)}^{AF}{}^E \times e^{-rf}) + (1 - p_{jE(t)}) \times (V_{T-t+1(j)}^{AF}{}^E \times e^{-rf}) +$$
$$C(0)_{T-t(i,j)} - C(IF^E)_{T-t(i,j)} \ (23)$$

In order to simplify this expression, we proceed to denote the option portfolio as $C(AF)^E{}_{T-t(i,j)} = C(0)_{T-t(i)} - C(IF^E)_{T-t(i,j)}$, the expression stays as,

$$V_{T-t(i,j)}^{AF}{}^E = C(AF)^E{}_{T-t(i,j)} + p_{jE(t)} \times (V_{T-t+1(i)}^{AF}{}^E \times e^{-rf}) + (1 - p_{jE(t)}) \times (V_{T-t+1(j)}^{AF}{}^E \times$$
$$e^{-rf}) \ (24)$$

2.3. Binomial Model, Liquidation Possibilities and Tax Savings Value

This section proposes the whole model: the binomial valuation model with liquidation possibilities (Broadie & Kaya, 2007; Milanesi, 2014) and tax savings determination through an option portfolio. Nevertheless, the highlight of this section is to consider results $EBIT^E$ and debt interest rate i^E are projected incorporating the Edgeworth transformation. Consequently, firm value, tax savings, debt and equity follow this process. The model's logic is the following: projected terminal results must be enough to cover debt payments. If not, we proceed to cancel debt interests and firm is liquidated. Firm value is $EBIT^E$ present value conditioned to liquidations scenarios and future tax savings present value. Shareholders' equity is equal to the addition between these two. On liquidations scenarios, we do not expect tax savings, and we assume that we cannot transfer tax losses to other individuals or firms. Then, we will present every step of the model:

a) *Projected unleveraged firm value*: we must project $EBIT^E$ using equation 8 being, on this case, the underlying asset result,

$$EBIT^E_{i,j,t} = EBIT_0 e^{\mu \times t + \sigma \sqrt{t} \times x_j} \ (25)$$

Next, we will project unleveraged firm value (V^E), after the deterministic result. As we want to add the project results (equation 25), we must deduct ratio $EBIT_0/V_o$, to firm value on $t=0$ (Brandao, Dyer & Hahn, 2005; Smith, 2005)[3], in order to use a present value adjusted by results $V_0{}^A$. As we want to simplify, we assume that this ratio stays constant through the whole projected period,

$$V_{i,j,t-1}{}^E = V_0{}^A e^{\mu \times t-1 + \sigma \sqrt{t-1} \times x_j} \ (26)$$

Finally, projected unleveraged firm value comes from adding obtained values on nodes (equations 25 and 26),

$$V^{E*}_{i,j,t} = V_{i,j,t}{}^E + EBIT^E_{i,j,t} \ (27)$$

b) *Binomial tree to estimate debt value*: we must start assuming that firm writes a bond with periodical payments (IF^E), composed by interest and principal cancelation at the end of its lifetime (P),

$$IF^E = i^E \times P \ (28)$$

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3 Results or projected cash flow are calculated through the traditional manner and are discounted from the start value in order to add results or obtained flows through the selected stochastic process.

Interest rate follows a transformed geometrical Brownian motion (equation 8) expressed in the following manner,

$$i^E_{i,j,t} = i_0 e^{\mu \times t + \sigma \sqrt{t} \times x_j} (29)$$

On this stage, interest is not adjusted by tax savings. Since it is contingent variable, we apply model contained on equations 22 and 23.

c) *Equity, debt, and asset value not conditioned to insolvency or continuity scenarios*: We must calculate equity value (E^E), debt (D^E), and unleveraged firm (V^{E*}) value without conditioning it to insolvency scenarios through a backward induction. This transformed binomial tree is the middle step for the final tree, but both start from terminal values (*T*), related to equity, debt, and total assets. They will be analytically presented on d). If results are higher than interests, then equity, debt and firm value come from equations 33, 34, 35 respectively. If not, we must apply equations 36, 37, and 38. From these terminal values, we must calculate the unconditioned value for moment *t=0* through backward induction. We must use implicit probabilities coefficient (equations 15 and 16),

$$\widetilde{E}_{(j,t)}{}^E = e^{-rf}(p_{jE(t)}\widetilde{E}_{(i,t+1)}{}^E + 1 - p_{jE(t)}\widetilde{E}_{(j,t+1)}{}^E) \; (30)$$

$$\widetilde{P}_{(j,t)}{}^E = e^{-rf}(p_{jE(t)}\widetilde{P}_{(i,t+1)}{}^E + 1 - p_{jE(t)}\widetilde{P}_{(j,t+1)}{}^E) \; (31)$$

$$\widetilde{V}_{(j,t)}{}^{E*} = e^{-rf}(p_{jE(t)}\widetilde{V}_{(i,t+1)}{}^E + 1 - p_{jE(t)}\widetilde{V}_{(j,t+1)}{}^E) \; (32)$$

Intermediate values are used on point d) which will be developed next.

d) *Firm value, equity, and tax savings conditioned to insolvency and continuity scenarios*: Once we have completed steps a, b, and c, we are ready to implement the final step of the model. There are three different situations depending on the time horizon: 1) Final horizon on tree projection (T), 2) intermediate nodes (*0<t<T*) and 3) initial moment (*t=1* →0). They are subject to the following condition I) *EBIT^E* higher than interest debt, then we assume continuity (*EBIT^E≥IF^E*); II) potential firm liquidation state as a financial default (*EBIT^E<IF^E*). These are the expressions for every step:

1) *Equations to estimate conditioned values at final time (T)*: These are equations that calculate terminal value explained on c) that enable us to apply equations 30, 31, and 32. They are the starting point and input to determine intermediate nodes:

I) Firm continuity (T): Firm value plus free cash flow is higher or equal to debt payment (interest plus capital) $V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E \geq IF_{(i,j)T}{}^E + P,$

$$E_{(i,j)T}{}^E = \left[V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E - IF_{(i,j)T}{}^E - P\right] + V^{AF}_{(i,j)T}{}^E \; (33)$$

$$P_{(i,j)T}{}^E = IF_{(i,j)t}{}^E + P \; (34)$$

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$$V_{(i,j)T}{}^E = \left[V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E\right] + V_{(i,j)T}^{AF}{}^E \quad (35)$$

II) Liquidation (T): Firm value plus free cash flow are insufficient to repay debt (interest plus capital) $V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E < IF_{(i,j)T}{}^E + P$. Transaction costs (α) are incorporated as part of the model solution, being,

$$E_{(i,j)T}{}^E = 0 \quad (36)$$

$$P_{(i,j)T}{}^E = (1 - \alpha)(V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E) \quad (37)$$

$$V_{(i,j)T}{}^E = (1 - \alpha)(V_{(i,j)T}{}^E + EBIT_{(i,j)T}{}^E) \quad (38)$$

Tax savings present value are not imputable once firm is over.

2) *Equations to estimate conditioned intermediate nodes value (t<T; t>0)*: Equations 33, 34, and 36 are the starting point. Intermediate values are obtained from: $\tilde{E}_{(i,j)t}{}^E$ (equation 30), $\tilde{P}_{(j,t)}{}^E$ (equation 31), and $\tilde{V}_{(j,t)}{}^{E*}$ (equation 32).

I) Firm continuity (t<T; t≥1): There is not firm liquidation risk if the addition between *EBIT^E* and shareholders' equity present value is enough to cover debt cash flows: $\tilde{E}_{(i,j)t}{}^E + EBIT_{(i,j)t}{}^E \geq IF_{(i,j)t}{}^E$, with the following group of equations, noted with the suffix c that indicates conditioning to the referred estates,

$$E_{(i,j)t,c}{}^E = \left[\tilde{E}_{(i,j)t}{}^E + EBIT_{(i,j)t}{}^E - IF_{(i,j)t}{}^E\right] + V_{T-t(i,j)}^{AF}{}^E \quad (39)$$

$$P_{(i,j)t,c}{}^E = IF_{(i,j)t}{}^E + \tilde{P}_{(j,t)}{}^E \quad (40)$$

$$V_{(i,j)t,c}{}^E = \left[EBIT_{(i,j)t}{}^E + \tilde{V}_{(j,t)}{}^{E*}\right] + V_{T-t(i,j)}^{AF}{}^E \quad (41)$$

On equations 39 and 41, tax savings is obtained through equations 22 and 23.

II) Liquidation (t<T; t>0): Firm liquidation occurs if the addition between *EBIT^E* and shareholders' equity present value is not enough to cover debt cash flows: $\tilde{E}_{(i,j)t}{}^E + EBIT_{(i,j)t}{}^E < IF_{(i,j)t}{}^E$. Default costs (α) represent a percentage of firm value,

$$E_{(i,j)t,c}{}^E = 0 \quad (42)$$

$$P_{(i,j)t,c}{}^E = (1 - \alpha)(\tilde{V}_{(j,t)}{}^{E*} + EBIT_{(i,j)t}{}^E) \quad (43)$$

$$V_{(i,j)t}{}^E = (1 - \alpha)(\tilde{V}_{(j,t)}{}^{E*} + EBIT_{(i,j)t}{}^E) \quad (44)$$

Tax savings present value does not compute as a result of firm liquidation and not transferring default values.

3) *Equations to estimate the initial conditioned value (t=1 →0)*: Finally, firm value and equity on *t=0* is obtained by intermediate values (*t<T; t>t=0*), These equations were

developed on the previous point. On this segment, we proceed to work from period *t=1* until *t=0*, (*t=1* →0). Equations are the following,

$$E_0{}^E = e^{-rf}\left\{\left[p_{jE(t1)}\left(E_{(i)t(1)}{}^E + EBIT_{(i)t(1)}{}^E - IF_{(i)t(1)}{}^E + V_{(i)t(1)}^{AF}{}^E\right)\right] + \left[1 - p_{jE(t1)}\left(\widetilde{E}_{(j)t(1)}{}^E + EBIT_{(j)t(1)}{}^E - IF_{(j)t(1)}{}^E + V_{(j)(1)}^{AF}{}^E\right)\right]\right\} \quad (45)$$

$$P_0{}^E = e^{-rf}\left(p_{jE(t1)}P_{(i)t(1)}{}^E + 1 - p_{jE(t1)}P_{(j)t(1)}{}^E\right) \quad (46)$$

$$V_0{}^E = e^{-rf}\left\{\left[p_{jE(t1)}\left(V_{(i)t(1)}{}^E + EBIT_{(i)t(1)}{}^E + V_{(i)(1)}^{AF}{}^E\right)\right] + \left[1 - p_{jE(t1)}\left(V_{(j)t(1)}{}^E + EBIT_{(j)t(1)}{}^E + V_{(j)t(1)}^{AF}{}^E\right)\right]\right\} \quad (47)$$

On this instance, we obtain expected firm value and equity conditioned to insolvency situations and incorporating tax savings.

3. Model Implementation: Hypothetical Case Analysis

In order to illustrate the proposed model, we will develop a hypothetical case scenario. We will try to use numbers as simplified as possible so we do not engross the objective. The objective is to estimate firm value (*V*), equity (*E*), and leverage (*P*) conditioned by continuity and liquidation scenarios. The latter will occur when firm does not have positive results in order to repay debt.

3.1. Assumptions and Initial Data

On *t=0*, we assume firm value applying discounted cash flow model is $1.000, with a result (*EBIT*) of $100 and weighted cost of capital (*k_o*) of 10%. Net firm value at initial estate (*V_0^A*) goes up to $900. Risk free rate (*rf*) is 5% annually, and cost of debt (*i*) is 8%. Debt ratio is 45% of firm value, thus leverage (*P*) is $450. It has the same behavior as a bullet bond, from *t=0* until *T-t* they only pay interests. On *T*, they repay debt. We assume that stochastic variables are expected results (*EBIT*), firm value (*V*), and cost of debt (*i*), having the same geometric Brownian motion since they share the same parameters. We assume that volatility (σ) of these previous variables is constant at 34%. It was obtained through the MAD method (Copeland & Antikarov, 2001) since it is a privately held company. Marginal earnings tax rate is 35%, and the firm is enabled to deduct interest tax savings from this imputable rent. In order to simplify the case, we assume that time horizon goes from t=0 until t=4. On the liquidation scenario, we fix transaction, default and dissolution costs (α) of 1%.

3.2. Projecting EBIT, i, and V

Our first step consists on estimating inputs to project stochastic variables: expected results, firm value, and financial cost of debt. We set up two different situations, asymmetry and mesokurtic curve (*E=0; K=3*); and negative asymmetry and platikurtic curve (*E=0.05; K=2.8*). This latter is set up to evidence non normal behavior through fat tails. Tables that show initial inputs (equations 1 to 16) to project variables and to apply backward processes with implicit probabilities are exposed on the appendix on tables A.1 to A.9. Then, we will present trees that we used in projecting initial stochastic variables. We must highlight that the financial cost of debt has a counter cyclical behavior to the results and firm value ones. So, on the up (*u*) scenario on EBIT and V projecting variable, cost of debt (*i*) diminishes as a result of a lower perception of firm financial risk. In contrast, on the down (*d*) scenarios, cost of debt rises.

EBIT E=0, K=3 (equation 25)					*EBIT E=-0.05 K=2.8 (equaiton 25)*				
0	1	2	3	4	0	1	2	3	4
$ 100.00	$ 141.22	$ 199.42	$ 281.61	$ 397.67	$ 100.00	$ 139.82	$ 198.92	$ 322.10	$ 1,532.57
	$ 70.13	$ 99.03	$ 139.84	$ 197.48		$ 69.39	$ 98.18	$ 151.86	$ 513.28
		$ 49.18	$ 69.44	$ 98.06			$ 48.46	$ 71.60	$ 171.90
			$ 34.48	$ 48.70				$ 33.75	$ 57.57
				$ 24.18					$ 19.28

Table 1. Projecting EBIT *(Source: Own elaboration)*

V E=0, K=3 (equation 26)					*V E=-0.05 K=2.8 (equation 26)*				
0	1	2	3	4	0	1	2	3	4
$ 900.00	$ 1,143.84	$ 1,615.28	$ 2,281.02	$ 3,221.15	$ 900.00	$ 1,132.57	$ 1,611.24	$ 2,608.98	$ 12,413.82
	$ 568.02	$ 802.12	$ 1,132.72	$ 1,599.57		$ 562.07	$ 795.26	$ 1,230.04	$ 4,157.57
		$ 398.32	$ 562.49	$ 794.33			$ 392.51	$ 579.92	$ 1,392.43
			$ 279.33	$ 394.45				$ 273.41	$ 466.34
				$ 195.88					$ 156.19

Table 2. Projecting V from V_0^A *(Source: Own elaboration)*

i E=0. K=3 (equation 29)					*i E=-0.05 K=2.8 (equation 29)*				
0	1	2	3	4	0	1	2	3	4
8.000%	5.610%	3.934%	2.759%	1.935%	8.000%	5.551%	3.877%	2.700%	1.543%
	11.297%	7.922%	5.555%	3.896%		11.186%	7.854%	5.728%	4.606%
		15.953%	11.187%	7.845%			15.914%	12.149%	13.752%
			22.529%	15.798%				25.768%	41.062%
				31.814%					122.606%

Table 3. Projecting cost of debt (i) *(Source: Own elaboration)*

3.3. Tax Savings as an Option Portfolio

We will determine tax savings which input is given on tables 1 and 3. Table 4 estimates a long call, in other words, operative earnings tax as long EBIT is higher than zero. Table 5 estimates a short call. Si, if results are higher than interests, then we must calculate interest tax savings. Otherwise, its value is zero.

long call (C(A)) E=0. K=3 (operative tax)					long call (C(A)) E=-0.05 K=2.8 (operative tax)				
0	1	2	3	4	0	1	2	3	4
$ 35.00	$ 49.43	$ 69.80	$ 98.56	$ 139.19	$ 35.00	$ 48.94	$ 69.62	$ 112.73	$ 536.40
	$ 24.54	$ 34.66	$ 48.94	$ 69.12		$ 24.29	$ 34.36	$ 53.15	$ 179.65
		$ 17.21	$ 24.31	$ 34.32			$ 16.96	$ 25.06	$ 60.17
			$ 12.07	$ 17.04				$ 11.81	$ 20.15
				$ 8.46					$ 6.75

Table 4. operative tax on the long call *(Source: Own elaboration)*

short call (C(i)) E=0. K=3					short call (C(i)) E=-0.05 K=2.8				
0	1	2	3	4	0	1	2	3	4
$ 12.60	$ 8.84	$ 6.20	$ 4.35	$ 3.05	$ 12.60	$ 8.74	$ 6.11	$ 4.25	2.43
	$ 17.79	$ 12.48	$ 8.75	$ 6.14		$ 17.62	$ 12.37	$ 9.02	7.25
		$ -	$ 17.62	$ 12.36			$ -	$ 19.13	21.66
			$ -	$ -				$ -	-
				$ -					-

Table 5. Tax savings conditioned to the existence of results higher than interests *(Source: Own elaboration)*

Through equation 21, we obtain the terminal value for each node since it is strike at the end of the exercise.

Tax savings (C(A)-C(f)) E=0. K=3. (equation 21)					Tax savings (C(A)-C(f)) E=-0.05 K=2.8 (equation 21)				
0	1	2	3	4	0	1	2	3	4
$ 12.60	$ 8.84	$ 6.20	$ 4.35	$ 3.05	$ 12.60	$ 8.74	$ 6.11	$ 4.25	2.43
	$ 17.79	$ 12.48	$ 8.75	$ 6.14		$ 17.62	$ 12.37	$ 9.02	7.25
		$ -	$ 17.62	$ 12.36			$ -	$ 19.13	21.66
			$ -	$ -				$ -	-
				$ -					-

Table 6. Option portfolio to estimating tax savings conditioned to the existence of results higher than interests *(Source: Own elaboration)*

Finally, we estimate tax savings present value in t=0 (equation 22 and 23) where the backward process uses implicit probabilities contained on table A.9 on the annex.

Tax savings present value E=0. K=3 (equation 22 y table A.9)					Tax savings present value E=-0.05 K=2.8 (equation 22 y table A.9)				
0	1	2	3	4	0	1	2	3	4
$ 145.40	$ 152.57	$ 137.63	$ 91.70	$ 60.94	$ 232.65	$ 291.11	$ 374.82	-$ 288.36	$ 48.59
	$ 126.64	$ 164.58	$ 184.65	$ 122.72		$ 191.68	$ 265.91	$ 320.23	$ 145.08
		$ 64.28	$ 135.16	$ 247.12			$ 107.97	$ 224.24	$ 433.20
			$ -	$ -				$ -	$ -
				$ -					$ -

Table 7. Tax savings present value *(Source: Own elaboration)*

If we would have applied traditional model (equation 19), without considering contingent value of results and interests, constant tax savings would have been $252. It drives us to overestimating them when the company faces a scenario with negative results or lower than the period interest. Table 7 presents tax savings value assuming a normal behavior ($145.4) and assuming a platikurtic curve ($232.65). Since results behavior related to the interest rate is counter cyclical, higher tax savings occur in middle nodes (lower results and higher interests).

3.4. Firm and Equity Value Conditioned by Continuity and Liquidation Scenarios

We will calculate firm and equity value conditioned by firm continuity or liquidation. On the first step, we will project stochastic variables (tables 1, 2, and 3). In order to calculate interests on every node, we used equation 28. Values are exposed on tables 8 and 9.

E=0, K=3 (equations 25, 26, 28, and 29)					
0	1	2	3	4	Concepts
$ 900.00	$ 1,143.84	$ 1,615.28	$ 2,281.02	$ 3,221.15	*V*
$ 100.00	$ 141.22	$ 199.42	$ 281.61	$ 397.67	*EBIT*
$ 36.00	$ 25.25	$ 17.70	$ 12.41	$ 458.71	*P*
	$ 568.02	$ 802.12	$ 1,132.72	$ 1,599.57	*V*
	$ 70.13	$ 99.03	$ 139.84	$ 197.48	*EBIT*
	$ 50.84	$ 35.65	$ 25.00	$ 467.53	*P*
		$ 398.32	$ 562.49	$ 794.33	*V*
		$ 49.18	$ 69.44	$ 98.06	*EBIT*
		$ 71.79	$ 50.34	$ 485.30	*P*
			$ 279.33	$ 394.45	*V*
			$ 34.48	$ 48.70	*EBIT*
			$ 101.38	$ 521.09	*P*
				$ 195.88	*V*
				$ 24.18	*EBIT*
				$ 521.09	*P*

Table 8. Projecting firm, results, and debt values (Source: Own elaboration)

E=-0.05 K=2.5 (equations 25, 26, 28, and 29)					
0	1	2	3	4	Concepts
$ 900.00	$ 1,132.57	$ 1,611.24	$ 2,608.98	$ 12,413.82	*V*
$ 100.00	$ 139.82	$ 198.92	$ 322.10	$ 1,532.57	*EBIT*
$ 36.00	$ 24.98	$ 17.45	$ 12.15	$ 456.94	*P*
	$ 562.07	$ 795.26	$ 1,230.04	$ 4,157.57	*V*
	$ 69.39	$ 98.18	$ 151.86	$ 513.28	*EBIT*
	$ 50.34	$ 35.34	$ 25.77	$ 470.73	*P*
		$ 392.51	$ 579.92	$ 1,392.43	*V*
		$ 48.46	$ 71.60	$ 171.90	*EBIT*
		$ 71.61	$ 54.67	$ 511.89	*P*
			$ 273.41	$ 466.34	*V*
			$ 33.75	$ 57.57	*EBIT*
			$ 115.95	$ 634.78	*P*
				$ 156.19	*V*
				$ 19.28	*EBIT*
				$ 634.78	*P*

Table 9. Projecting firm, results, and debt values with asymmetry and kurtosis *(Source: Own elaboration)*

On the second step, we will estimate intrinsic values without any conditioning to continuity or liquidation scenarios. Again, on the backward process, we will use implicit probabilities as part of the transformation (Appendix table A.9)

E=0, K=3 (equations 30, 31, 32, and table A.9)					
0	1	2	3	4	Concepts
$ 662.93	$ 1,026.44	$ 1,533.64	$ 2,222.69	$ 3,221.06	*E*
$ 369.11	$ 404.09	$ 425.06	$ 440.53	$ 458.71	*P*
$ 1,032.04	$ 1,430.53	$ 1,958.71	$ 2,663.22	$ 3,679.76	*V*
	$ 367.40	$ 624.49	$ 1,001.86	$ 1,452.24	*E*
	$ 371.98	$ 424.56	$ 453.18	$ 467.53	*P*
	$ 739.38	$ 1,049.05	$ 1,455.04	$ 1,919.77	*V*
		$ 147.99	$ 311.15	$ 654.21	*E*
		$ 357.55	$ 439.48	$ 485.30	*P*
		$ 505.53	$ 750.63	$ 1,139.51	*V*
			$ -	$ -	*E*
			$ 312.28	$ 438.72	*P*
			$ 312.28	$ 438.72	*V*
				$ -	*E*
				$ 217.86	*P*
				$ 217.86	*V*

Table 10. Firm, equity, and debt value *(Source: Own elaboration)*

0	1	2	3	4	Conceptos
E=-0.05 K=2.8 (equations 30, 31, 32, and table A.9)					
$ 586.94	$ 348.12	-$ 2,205.84	$ 45,158.23	$ 13,538.04	E
$ 396.35	$ 436.40	$ 458.55	$ 386.25	$ 456.94	P
$ 983.29	$ 784.52	-$ 1,747.29	$ 45,544.48	$ 13,994.98	V
	$ 795.45	$ 1,386.12	$ 2,414.24	$ 4,345.20	E
	$ 403.58	$ 458.86	$ 472.51	$ 470.73	P
	$ 1,199.03	$ 1,844.98	$ 2,886.75	$ 4,815.93	V
		$ 338.68	$ 703.41	$ 1,485.65	E
		$ 392.98	$ 490.17	$ 511.89	P
		$ 731.66	$ 1,193.58	$ 1,997.53	V
			$ -	$ -	E
			$ 334.16	$ 518.68	P
			$ 334.16	$ 518.68	V
				$ -	E
				$ 173.71	P
				$ 173.71	V

Table 11. Firm, equity, and debt value with asymmetry and kurtosis *(Source: Own elaboration)*

On the third step, we will calculate present value conditioned to the proposed scenarios. The backward induction process with the transformation coefficient (appendix table A.9) is applied from *t=1 →0.*

t=0	1<t<T			T	Concepts
0	1	2	3	4	
E=0, K=3 (equations 33, 34, 35, 36, 37, 38, 39, 40, 41 and table A.9)					
$ 1,057.19	$ 1,294.98	$ 1,852.99	$ 2,583.58	$ 3,221.06	E
$ 441.49	$ 429.34	$ 442.77	$ 452.95	$ 458.71	P
$ 1,498.68	$ 1,724.32	$ 2,295.75	$ 3,036.52	$ 3,679.76	V
$ 1,498.68	$ 513.33	$ 852.45	$ 1,301.35	$ 1,452.24	E
	$ 422.82	$ 460.21	$ 478.18	$ 467.53	P
	$ 936.15	$ 1,312.66	$ 1,779.54	$ 1,919.77	V
		$ 189.66	$ 465.41	$ 654.21	E
		$ 429.34	$ 489.82	$ 485.30	P
		$ 618.99	$ 955.23	$ 1,139.51	V
			$ -	$ -	E
			$ 311.02	$ 438.72	P
			$ 310.67	$ 438.72	V
				$ -	E
				$ 217.86	P
				$ 217.86	V

Table 12. Conditioned firm, equity, and debt value *(Source: Own elaboration)*

E=-0.05 K=2.8 (equations 33, 34, 35, 36, 37, 38, 39, 40, 41 and table A.9)					
t=0	1<t<T			T	Concepts
0	1	2	3	4	
$ 1,130.51	$ 754.07	$ -	$ 45,179.81	$ 13,538.04	E
$ 471.15	$ 461.38	$ 1,794.05	$ 398.40	$ 456.94	P
$ 1,601.66	$ 1,215.45	$ 1,792.06	$ 45,578.22	$ 13,994.98	V
$ 1,601.66	$ 1,006.19	$ 1,714.86	$ 2,860.56	$ 4,345.20	E
	$ 453.92	$ 494.21	$ 498.29	$ 470.73	P
	$ 1,460.10	$ 2,209.07	$ 3,358.84	$ 4,815.93	V
		$ 423.49	$ 944.58	$ 1,485.65	E
		$ 464.59	$ 544.83	$ 511.89	P
		$ 888.08	$ 1,489.42	$ 1,997.53	V
			$ -	$ -	E
			$ 304.43	$ 518.68	P
			$ 304.09	$ 518.68	V
				$ -	E
				$ 173.71	P
				$ 173.71	V

Table 13. Conditioned firm, equity, and debt value with asymmetry and kurtosis *(Source: Own elaboration)*

On table 14, we summarize values and, as it occurs with the traditional discounted cash flow method, it underestimates firm (*V*) and equity value (*E*) assuming unconditioned constant behaviors. Moreover, it overestimates tax savings since it assumes that is unconditioned. The numerical model adapts itself to different continuity or liquidation scenarios as the company could be able to cancel interests depending on the results.

Value	DCF	OR E=0, K=3	OR E=-0.05, K=3
V	$ 1,252.00	$ 1,498.68	$ 1,601.66
E	$ 802.00	$ 1,057.19	$ 1,130.51
P	$ 450.00	$ 441.49	$ 471.15

Table 14. DCF and binomial model summary *(Source: Own elaboration)*

In order to approximate better to business stochastic behavior characterized by its dynamism and turbulence, it is valid to assume that variables would adopt fat tails biases on the distribution curve. Based on this idea, the proposed model captures asymmetry and kurtosis expressed on values of tables 13 and 14. Consequently, the wide range of expected values that explain present value is higher while incorporating potential positive and negative events.

4. Conclusion

Companies live on a turbulent environment full of improbable events that are not captured by the mean behavior found on the normal probability distribution. Under these circumstances, traditional valuation models like the discounted cash flow model are useless to value ongoing companies; in particular, closed companies that operate on emerging contexts or growing segments. Intrinsic value generated by the discounted cash flow model assumes lineal and growing behaviors unconditioned to contingent scenarios of losses and earnings. Consequently, it impacts on firm and equity value and, eventually, on how we must treat tax savings.

As a first solution of this problem, we developed an experimental numerical model which is based under a binomial distribution. Its main contribution consists on the treatment of firm and equity value related to potential continuity and liquidation scenarios. Moreover, it assumes the existence of tax savings avoiding the debate under which rate should be discounted. On both cases, it adopts real option theory to value tax savings as a result of leverage and determining firm value incorporating solvency and dissolution scenarios. This model complements the tradition binomial stochastic process with the Edgeworth transformation. It allows incorporating asymmetry and kurtosis abandoning the assumption of normal behavior. The presented example assumes a counter cyclical behavior among results, firm value and debt cost. At the same time, results are analyzed under the traditional version and with higher moments. On the latter, the magnitude of potential values is higher since the model abandons dispersions that are only explained by the standard deviation. Thus, our model fulfills the objective of approaching a more realistic behavior.

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APPENDIX

Random Variable xj (equation 2)				Associated Probability b(x) (equation 3)			
1	2	3	4	1	2	3	4
			-2.00				0.0625
		-1.73				0.125	
	-1.41		-1.00		0.25		0.25
-1.00		-0.58		0.5		0.375	
	0.00		0.00		0.5		0.375
1.00		0.58		0.5		0.375	
	1.41		1.00		0.25		0.25
		1.73				0.125	
			2.00				0.0625

Table A.1 *(Source: Own elaboration)*

Edgeworth Expansion W(x) with E=0 y K=3 (equation 4)					Edgeworth Expansion W(x) with E=-0.05 y K=2.8 (equation 4)				
0	1	2	3	4	0	1	2	3	4
				1					0.974
			1					0.974	
		1		1			0.974		1.034
	1		1			0.974		1.034	
1		1		1	0.974		1.034		1.025
	1		1			1.034		1.025	
		1		1			1.025		0.597
			1					0.597	
				1					-0.758

Table A.2 *(Source: Own elaboration)*

g(x)=b(x), w(x) with E=0, K=3 (eq.3 x eq.4)					g(x)=b(x), w(x) with E=-0.05 K=2.8 (eq.3 x eq.4)				
0	1	2	3	4	0	1	2	3	4
				0.0625					0.06
			0.125					0.12	
		0.25		0.25			0.24		0.26
	0.5		0.375			0.49		0.39	
0		0.5		0.375	0.00		0.52		0.38
	0.5		0.375			0.52		0.38	
		0.25		0.25			0.26		0.15
			0.125					0.07	
				0.0625					-0.05
	1	1	1	1		1.00	1.02	0.97	0.81

Table A.3 *(Source: Own elaboration)*

Weighted corrected probabilities E=0, K=3					Weighted corrected probabilities E=-0.05, K=2.8				
0	1	2	3	4	0	1	2	3	4
				0.0625					0.08
			0.125					0.13	
		0.25		0.25			0.24		0.32
	0.5		0.375			0.49		0.40	
0		0.5		0.375	0		0.51		0.48
	0.5		0.375			0.51		0.40	
		0.25		0.25			0.25		0.19
			0.125					0.08	
				0.0625					-0.06
	1	1	1	1		1	1	1	1

Table A.4 *(Source: Own elaboration)*

Parameters for standardization with E=0 and K=3				Parameters for standardization with E=-0.05 y K=2.8			
Mean (equation 5)				Mean (equation 5)			
1	2	3	4	1	2	3	4
			0				-0.40466
		0				-0.08655	
	0				0.01744		
0				0.02958			
Variance (ecuación 6)				Variance (equation 6)			
1	2	3	4	1	2	3	4
			1				0.40950
		1				0.86668	
	1				0.98281		
1				0.99912			
Deviation				Deviation			
1	2	3	4	1	2	3	4
			1				0.63992
		1				0.93096	
	1				0.99137		
1				0.99956			

Table A.5 *(Source: Own elaboration)*

Variable xj standardized E=0. K=3 (equation 7)				Variable xj standardized E=-0.05 K=2.8 (equation 7)			
1	2	3	4	1	2	3	4
			-2.000				-2.4930
		-1.732				-1.7675	
	-1.414		-1.000		-1.4441		-0.9303
-1.000		-0.577		-1.0305		-0.5272	
	0.000		0.000		-0.0176		0.6324
1.000		0.577		0.9713		0.7131	
	1.414		1.000		1.4089		2.1950
		1.732				1.9535	
			2.000				3.7577

Table A.6 *(Source: Own elaboration)*

μ (equation 12) E=0, K=3					μ (equation 12) E=-0.05 K=2.8				
0	1	2	3	4	0	1	2	3	4
				-0.004885					0.024781
			-0.004885					-0.004849	
		-0.004885					-0.004831		
	-0.004885					-0.004733			
0.000000					0.000000				

Table A.7 *(Source: Own elaboration)*

Implicit probabilities (equation 14) E=0. K=3					
0	1	2	3	4	Nodes
1	0.5000	0.2500	0.1250	0.0625	0
	0.5000	0.2500	0.1250	0.0625	1
		0.2500	0.1250	0.0625	2
			0.1250	0.0625	3
				0.0625	4
Implicit probabilities (equation 14) E=-0.05 K=2.8					
0	1	2	3	4	Nodes
1	0.398836	0.113273	-0.012543	-0.058846	0
	0.601164	0.285563	0.125816	0.046303	1
		0.315601	0.159746	0.079513	2
			0.155855	0.080233	3
				0.075622	4

Table A.8 *(Source: Own elaboration)*

Certain equivalent coefficients (equations 15 y 16) E=0, K=3					Certain equivalent coefficients (equations 15 y 16) E=-0.05 K=2.8				
1	2	3	4	Nodes	1	2	3	4	Nodes
0.5	0.5	0.5	0.5	0	0.3988	0.2840	-0.1107	4.6915	0
0.5	0.5	0.5	0.5	0	0.6012	0.7160	1.1107	-3.6915	0
	0.5	0.5	0.5	1		0.4750	0.4406	0.3680	1
	0.5	0.5	0.5	1		0.5250	0.5594	0.6320	1
		0.5	0.5	2			0.5062	0.4977	2
		0.5	0.5	2			0.4938	0.5023	2
			0.5	3				0.5148	3
			0.5	3				0.4852	3

Table A.9 *(Source: Own elaboration)*