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Ten years of compositional analysis of financial ratios. Review, guidelines, and future directions

Germà Coenders
Department of Economics. Universitat de Girona, Girona, Spain
Corresponding author. Facultat d'Economia i Empresa. C. Universitat de Girona 10. 17003 Girona, Spain. germa.coenders@udg.edu
https://orcid.org/0000-0002-5204-6882

Núria Arimany-Serrat
Department of Economics and Business. Universitat de Vic-Universitat Central de Catalunya, Vic, Spain.
https://orcid.org/0000-0003-0323-6601

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ABSTRACT

Compositional data are contemporarily defined as positive vectors, the ratios among whose elements are of interest to the researcher. Financial statement analysis by means of accounting ratios a.k.a. financial ratios fulfils this definition to the letter. Compositional data analysis solves the major problems in statistical analysis of standard financial ratios, such as skewness, non-normality, non-linearity, outliers, and dependence of the results on the choice of which accounting figure goes to the numerator and to the denominator of the ratio. In this article, we review the literature on said problems and the applications of the compositional data methodology since the first publication 10 years ago. We present a how-to guide for early adopters of the methodology. This includes some transformations within compositional data analysis that are particularly useful for financial statement analysis, how to visualise firms in an industry with a compositional principal-component-analysis biplot and how to do statistical inference on the financial performance by groups of firms. We discuss in which cases compositional data analysis is essential and in which it can be dispensed with, as well as current debates, future challenges and avenues for further research.

KEYWORDS

CoDa, log-ratios, margin, turnover, leverage, ROE, SABI

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1. Introduction and literature review

The relative nature of *financial ratios*, a.k.a. *accounting ratios* (i.e., ratios comparing selected figures in financial statements), enables them to evaluate the firm's position compared to its counterparts in the industry or to itself over time, taking into account differences or evolution in firm size (e.g., Ross et al., 2003). Financial ratios have been used in practical management performance and strategic assessment and in research relating them to other financial or non-financial variables (Altman, 1968; Amat Salas, 2020; Barnes, 1987; Faello, 2015; Horrigan, 1968; Ros et al., 2019; Soukal et al., 2024; Staňková and Hampel, 2023; Veganzones and Severin, 2021; Willer do Prado et al., 2016) including, for instance, fraud detection, stock market returns, and firm survival, default or bankruptcy. A typical example of financial ratio is that of assets over equity. This ratio tells how indebted a firm is, since assets can be decomposed into equity and liabilities, i.e., debt. It is also referred to as the leverage ratio.

While the use of standard financial ratios in diagnostics of the financial health of single firms (or comparing, e.g., two firms) is straightforward, analysing a sample of firms from an industry requires statistical methods. Using standard financial ratios as variables in statistical analyses has been related to a number of serious problems, such as *asymmetry* (Faello, 2015; Frecka and Hopwood, 1983; Linares-Mustarós, et al., 2018; Oktaviano et al., 2024), connected to it severe *non-normality* of the distributions (Buijink and Jegers, 1986; Deakin, 1976; Durana et al., 2025; Iotti et al., 2023; 2024a; 2024b; 2024c; Lueg et al., 2014; Martikainen et al., 1995; McLeay and Omar, 2000; So, 1987; Valaskova et al., 2023), *non-linearity* of the relationships (Balcaen and Ooghe, 2006; Carreras-Simó and Coenders, 2021; Cowen and Hoffer, 1982; Keasey and Watson, 1991; Taoushianis, 2025), extreme *outliers* (Deshpande, 2023; Ezzamel and Mar-Molinero, 1990; Frecka and Hopwood, 1983; Kane et al., 1998; Lev and Sunder, 1979; Liu et al., 2025; Magrini, 2025; McLeay, 1986; Nyitrai and Virág, 2019; Oktaviano et al., 2024; Watson, 1990), and even dependence of the results on the arbitrary decision regarding which accounting figure appears in the numerator and which in the denominator of the ratio (Coenders and Arimany-Serrat, 2025; Coenders et al., 2023a; Frecka and Hopwood, 1983; Linares-Mustarós, et al., 2022). The results of many statistical analyses are invalid when all or some of these problems occur and the results and conclusions of said analyses can be affected to a great extent. It has long been known that these problems not only have serious consequences for single ratios but also when combining ratios into composite indicators by means of factor analysis and related methods (Cowen and Hoffer, 1982; Martikainen et al., 1995). Readers unfamiliar with the statistical consequences of asymmetry, outliers, non-linearity, and so on, can resort to any introductory statistics or econometrics handbook.

These serious problems have also been reported in other scientific fields using ratios (Isles, 2020). It must be noted that when financial ratios were first developed in the late 19th century (Brown Sister, 1955; Horrigan, 1968), statistical analysis methods were in their infancy, which speaks by itself for the fact that financial ratios were not conceived

with a statistical usage in mind. The theory on *ratio measurement scales* (Stevens, 1946) had not even been developed. This situation has changed dramatically, with a large body of statistical and econometrical research in the field of accounting being available nowadays (Gruszczyński, 2022). As a matter of fact, the first use of the term *econometrics* was made by Paweł Ciompa in 1910 in the accounting field (Ciompa, 1910). The aim of this article is to review the solutions presented to these problems and present one of them, which is called *Compositional Data* (CoDa) approach.

Until recently, only rarely has the accounting research community recognised the importance of these problems, and when this has been the case, diverse ad-hoc solutions have been applied on the spot for a problem at a time. For instance, outliers have been dealt with by just removing them, which is called trimming or pruning (Ezzamel and Mar-Molinero, 1990; Frecka and Hopwood, 1983; Kubenka and Myskova, 2022; Martikainen et al., 1995; Magrini, 2025; So, 1987; Watson, 1990) or replacing them with the nearest non-outlying values, which is called wisorizing (Demiraj et al., 2024; Deshpande, 2023; Duong et al., 2025; Gupta, 2024; Lev and Sunder, 1979; Laus et al., 2025; Liu et al., 2025; Naz et al., 2023; Nyitrai and Virág, 2019; Vu et al., 2023). The number of identified outliers in at least one ratio has in some cases neared one third of all firms, and these procedures have thus seriously impaired the sample representativeness (Martin-Melero et al., 2025). Asymmetry has been dealt with by using hard-to-interpret transformations such as the square root, the cubic root or generalised polynomials (Akbar et al., 2025; Deakin, 1976; Ezzamel and Mar-Molinero, 1990; Frecka and Hopwood, 1983; Martikainen et al., 1995; Taoushianis, 2025), their generalization as Box-Cox transformations (Mcleay and Omar, 2000; Watson, 1990), or by ignoring the original ratio values and considering only their rank order (Cheng and Fang, 2025; Kane et al., 1998; Lueg et al., 2014). Non-normality has been dealt with non-parametric statistics (Durana et al., 2025; Hazami-Ammar, 2024; Iotti et al., 2023; 2024a; 2024b; 2024c; Latief and Suhendah, 2023; Valaskova et al., 2023). Rather than that, here we present a simple unified approach to deal with all problems simultaneously which makes it possible to use the full sample without removing outliers and is compatible with any statistical method and not just with non-parametric methods.

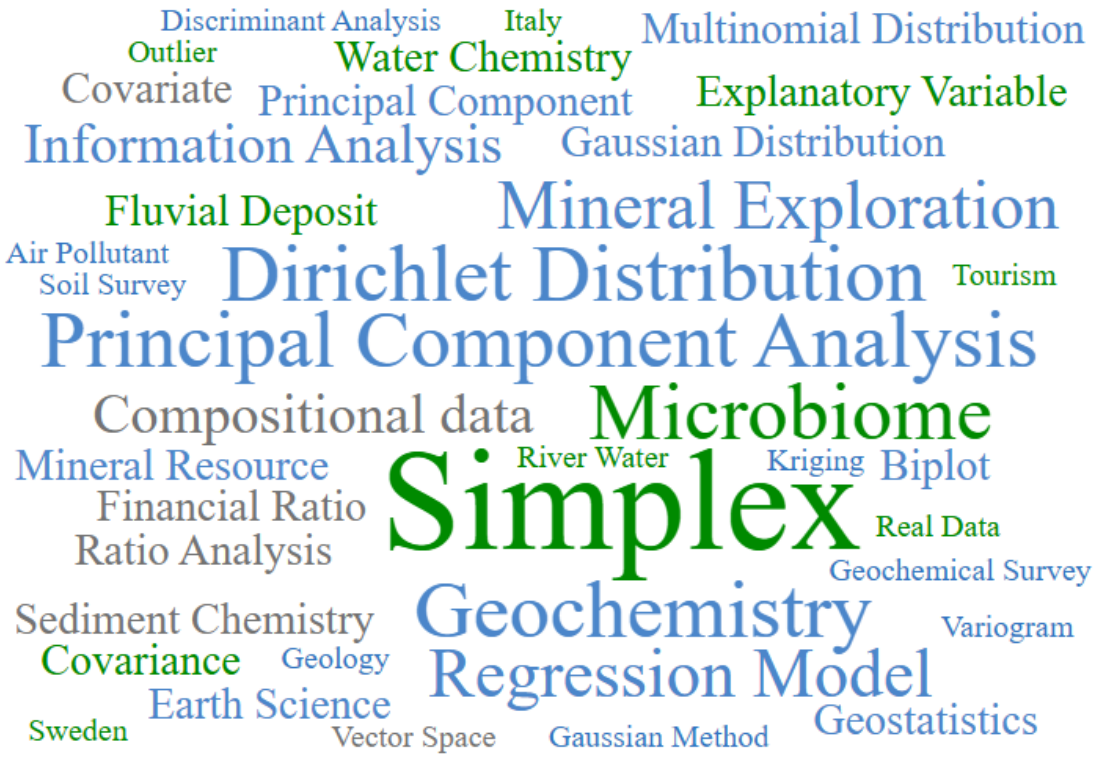
Financial ratios constitute a genuine case of researchers' and professionals' interest in relative rather than absolute magnitudes of accounting figures and thus a natural field of application of CoDa analysis, which has the same objective. Essentially, CoDa are arrays of strictly positive numbers for which ratios between them are considered to be relevant (Egozcue and Pawlowsky-Glahn, 2019) which perfectly fits the notion of financial statement analysis. The CoDa methodology offers a number of advantages in statistical analysis of financial statements, as compared to standard financial ratios. Among other features, CoDa treat accounting figures in a symmetric fashion in such a way that results do not depend on numerator and denominator permutation. CoDa also tend to reduce outliers, asymmetry, and non-normality, and to linearize relationships. Far from being a statistical refinement, the CoDa methodology leads to very substantial differences in the analysis results whenever it has been compared with standard financial ratios, like clusters composed only of outliers under the standard approach (Dao et al., 2024; Hernandez-Romero and Coenders, (2025); Jofre-Campuzano and Coenders, 2022; Linares-Mustarós et al., 2018), variables whose effect is statistically significant under the standard approach but not the compositional approach, or vice-versa (Arimany-Serrat et al., 2022; Carreras-Simó and Coenders, 2021; Coenders et al., 2023a; Creixans-Tenas et al., 2019; Escaramís and Arbussà, 2025; Linares-Mustarós et al., 2022), and substantial differences in forecasts

(Magrini, 2025).

Since the seminal works by Aitchison (1982, 1986), CoDa analysis has become a well-established methodology, notably present in accessible textbooks (van den Boogaart and Tolosana-Delgado, 2013; Filzmoser et al., 2018; Greenacre, 2018; Pawlowsky-Glahn et al., 2015) and software (Comas-Cufí and Thió-Henestrosa, 2011; van den Boogaart and Tolosana-Delgado, 2013; Filzmoser et al., 2018; Greenacre, 2018; Palarea-Albaladejo and Martín-Fernández, 2015; Thió-Henestrosa and Martín-Fernández, 2005), and continues to be further developed well after forty years (Coenders et al., 2023b; Greenacre et al., 2023).

The study of the relative importance of chemical elements in geological analysis spurred most of the early interest in CoDa (Aitchison, 1986; Buccianti et al., 2006). Nowadays, CoDa are being used in the social sciences and economics in general (Coenders and Ferrer-Rosell, 2020; Fry, 2011; Martínez-Garcia et al., 2023) and in finance in particular, to answer research questions concerning the relative importance of magnitudes (see Arimany-Serrat and Coenders, 2025; Arimany-Serrat et al., 2022; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Linares-Mustarós et al., 2022 for reviews on the topic). Examples include crowdfunding, bond ratings, financial markets, municipal budgeting, insurance, exchange rates, banking, portfolios, systemic risk, household finance, intermediary market share, and equity ownership structure. The word cloud of the CoDa topic according to the SCOPUS data base shows “Financial ratio” to be a leading subtopic (Figure 1), surrounded by mathematical, statistical, geological, chemical, biological and medical concepts.

Figure 1. SCOPUS word cloud of the CoDa topic (accessed 19/09/2025)



The foundations of CoDa in financial-ratio analysis were laid for the first time in Linares-

Mustarós and Coenders (2016). Since then, the method has been applied in the industries of manufacture of pharmaceutical preparations (Linares-Mustarós et al., 2018), manufacture of wearing apparel (Linares-Mustarós et al., 2018; Molas-Colomer et al., 2024), hospitals (Creixans-Tenas et al., 2019), groceries (Carreras-Simó and Coenders, 2020), retail trade (Carreras-Simó and Coenders, 2021), wineries (Arimany-Serrat et al., 2022; 2023; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Linares-Mustarós et al., 2022), breweries (Arimany-Serrat and Sgorla, 2024; Coenders and Arimany-Serrat, 2025; Coenders et al., 2023a), hospitality (Farreras-Noguer et al., 2025; Mulet-Forteza et al., 2024; Saus-Sala et al., 2021; 2023; 2024), beekeeping (Arimany-Serrat and Coenders, 2025), fisheries (Dao et al., 2024), food and beverage processing (Dao et al., 2024; Rondós-Casas et al., 2026), grain and fat cultivation and processing (Hernandez-Romero and Coenders, 2025), and sale of automotive fuel (Jofre-Campuzano and Coenders, 2022). It has also been applied to more general populations, such as family firms (Escaramís and Arbussà, 2025) or even a country's economy at large (Magrini, 2025). To the best of our knowledge, these are all CoDa applications to financial ratios published so far in any academic format (articles, books, book chapters, and preprints), the search not being restricted to any particular data base. To avoid duplication, conference proceedings have been excluded when there is a corresponding full article of the same research work.

One of the strong points of the CoDa approach is that it provides a unified approach which suits itself to any statistical analysis. After log-ratio transformation, any statistical method can be applied in a standard manner and then interpreted from a compositional perspective. The range of statistical methods used so far includes regression models (Arimany-Serrat et al., 2023; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Farreras-Noguer et al., 2025; Mulet-Forteza et al., 2024), principal component analysis and biplots (Carreras-Simó and Coenders, 2020; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Rondós-Casas et al., 2026; Saus-Sala et al., 2021; 2023), cluster analysis (Arimany-Serrat and Coenders, 2025; Arimany-Serrat and Sgorla, 2024; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Hernandez-Romero and Coenders, 2025; Linares-Mustarós et al., 2018; Saus-Sala et al., 2021; 2023; 2024), weighted clustering (Dao et al., 2024; Jofre-Campuzano and Coenders, 2022), fuzzy clustering (Molas-Colomer et al., 2024), logit models (Magrini, 2025), random forests (Magrini, 2025), partial-least-squares structural equation modelling (Creixans-Tenas et al., 2019), vector autoregressive models (Carreras-Simó and Coenders, 2021), and panel regression models (Escaramís and Arbussà, 2025).

The compositional methodology has potential for any business research project using statistical models and including financial ratios among the set of study variables. Carreras-Simó and Coenders (2021) relate asset and capital structures, Escaramís and Arbussà (2025) compare capital structures of family and non-family firms, Magrini (2025) predicts bankruptcy, Creixans-Tenas et al. (2019) study the impact of social responsibility on profitability and solvency, Mulet-Forteza et al. (2024) the impact of expansion strategies, Farreras-Noguer et al. (2025) the impact of subsidies, Coenders (2025), and Linares-Mustarós et al. (2022) the impact of branding, Coenders et al. (2023a) the impact of non-financial reporting, Hernandez-Romero and Coenders (2025) the impact of the Ukraine war, and Arimany-Serrat et al. (2023), Arimany-Serrat and Sgorla, (2024), Dao et al. (2024), and Saus-Sala et al. (2024) the impact of Covid-19.

After this literature review, this article presents a simple how-to guide to motivate the use

of the CoDa approach to financial statement analysis. It starts explaining why and how financial statements and financial ratios should be considered as CoDa, including the necessary transformations. *DuPont analysis*, a very simple case of financial statement analysis is used as storyline. Then, the dataset of an example in the Spanish winery industry is presented. Next, financial statements are summarised and visualised. Next, the equality of financial structures is tested by groups of firms. The final section concludes.

2. Financial statements as compositional data

A D–part composition is defined as an array $\mathbf{x}$ of D strictly positive numbers x_j, called *parts*, the relative magnitude of which is of interest to the researcher (Aitchison, 1986):

$$\mathbf{x} = (x_1, x_2, ..., x_D) \text{ with } x_j > 0,\ j = 1, 2, ..., D\ , \tag{1}$$

Some rules have to be followed in order to introduce accounting figures in financial statements in a D–part composition, which boil down to avoiding negative accounting figures and their overlap (Creixans-Tenas et al., 2019):

- Even if sometimes financial ratios involve accounting figures which may be negative, its use is advised against in the financial literature, because they can cause a discontinuity, outliers, or even a reversal of interpretation when the accounting figure which may be negative is in the denominator (Lev and Sunder, 1979; Linares-Mustarós et al., 2022). Negative accounting figures are also advised against from the point of view of *measurement theory*. Computing a ratio is a meaningful operation only for variables in a *ratio scale*, which need to have a meaningful absolute zero (Stevens, 1946) and thus no negative values.
  In general, accounting figures are negative because they imply the subtraction of other positive accounting figures, which are the ones to be used. This means, for instance, that when building ratios, one should directly use revenues and costs rather than profit or current assets and current liabilities rather than working capital. This limitation implies no loss of information whatsoever. For instance, a ratio conveying the same information as the standard *margin* ratio (profit/revenues) can be constructed from only the non-negative revenue and cost figures. Let x_1=revenues, x_2=costs, x_3=x_1–x_2=profit. The always positive revenues over costs ratio (x_1/x_2) can easily be shown to be just a transformation of the problematic profit over revenues ratio (x_3/x_1):

$$\frac{x_1}{x_2} = \frac{x_1}{x_1 - x_3} = \frac{1}{\frac{x_1 - x_3}{x_1}} = \frac{1}{1 - \frac{x_3}{x_1}}\ . \tag{2}$$

- It must also be taken into account that parts may not overlap (Creixans-Tenas et al., 2019). For instance, one could not use x_4: assets and x_5: non-current assets because x_5 is a part of x_4. In compositional data terminology, x_4: assets are an *amalgamation* of x_5: non-current assets and x_6: current assets. Using both amalgamations and their constituent parts is extremely problematic (Pawlowsky-Glahn et al., 2015). Rather, the choice between using only the amalgamation or only the individual parts should be made at the problem definition stage and cannot be changed afterwards (van den Boogaart and Tolosana-Delgado, 2013).

> It is not essential to use all constituent parts, which is referred to as a *subcomposition* in compositional data terminology. Accordingly, the feasible choices to handle x_4 to x_6 are: a) to use only x_4; b) to use x_5 and x_6; c) to use only x_5; and d) to use only x_6.

The ultimate choice of parts will depend on the analysis objectives or research questions. The researchers will in principle like to select the accounting figures needed to compute their favourite standard financial ratios and refine the choice by avoiding overlap and subtraction. In the example we use in this article, the parts represented by the x_j variables are the following D=4 positive and non-overlapping financial statement account categories:

- x_1: revenues,
- x_2: costs,
- x_3: liabilities,
- x_4: assets.

These account categories are very relevant in practice because they make it possible to compute the common *turnover*, margin, and *leverage* ratios in classical DuPont analysis (Saus-Sala et al., 2021; 2023). Return on equity can be decomposed as the product of the three.
Turnover measures the efficiency of a firm's use of its assets in generating sales revenue:

$$\text{Turnover}=\text{revenues}/\text{assets}=x_1/x_4. \quad (3)$$

Margin is the percentage of sales revenue that is turned into profit:

$$\text{Margin}=\text{profit}/\text{revenues}=(x_1-x_2)/x_1. \quad (4)$$

Leverage measures assets generated per unit of shareholders equity. It is also a measure of indebtedness:

$$\text{Leverage}=\text{assets}/\text{equity}=x_4/(x_4-x_3). \quad (5)$$

DuPont analysis is chosen as storyline in this article both because of its simplicity (with only D=4 account categories involving just a handful of ratios) and its popularity. It goes without saying that account categories could be considered in greater detail, by increasing both the number of accounting figures D and the set of feasible financial ratios computed from them, which would make for a more detailed financial statement analysis.

3. CoDa transformations

3.1. Pairwise log-ratios

The usual approach to statistical analysis of CoDa is to use existent standard statistical methods on transformed data. *Logarithms of ratios* are the standard transformation in CoDa (Aitchison, 1986). The simplest case of a log-ratio is that between only two accounting figures which is called *pairwise log-ratios* (plr) in Coenders and Arimany-Serrat (2025); Creixans-Tenas et al. (2019); Farreras-Noguer et al. (2025); Greenacre

(2018; 2019); Mulet-Forteza et al. (2024); and Saus-Sala et al. (2021). It can also be understood as the log difference between the two accounting figures:

$$\log\left(\frac{x_1}{x_2}\right) = \log\left(x_1\right) - \log\left(x_2\right). \tag{6}$$

Unlike a standard ratio, which is bounded between zero and infinity, a plr is symmetric in the sense that its range is from minus infinity to plus infinity (Linares-Mustarós et al., 2018). This has two key advantages.

- On the one hand it coincides with the support of the normal probability distribution. It is clear that a bounded variable cannot possibly be normally distributed. Although there is never the guarantee that an unbounded real variable will be normally distributed, it is often the case for plr in empirical data sets (Aitchison, 1982).
- On the other hand, when predicting a standard ratio in a statistical model, some predicted values could actually be impossible negative values. The prediction for a plr will never be an impossible value.

Besides, a plr is symmetric in the sense that permuting the numerator and denominator parts leads to the same distance from zero and affects no other property of the plr than the sign (Linares-Mustarós et al., 2022):

$$\log\left(\frac{x_1}{x_2}\right) = \log\left(x_1\right) - \log\left(x_2\right) = -\left(\log\left(x_2\right) - \log\left(x_1\right)\right) = -\log\left(\frac{x_2}{x_1}\right). \tag{7}$$

For instance, the Pearson correlation coefficient of an external non-financial indicator with a permuted plr equals the correlation with the original plr with a reversed sign. This property does not hold for standard financial ratios. Correlating x_1/x_2 with a non-financial indicator can give conflicting results with respect to correlating it with x_2/x_1 (Coenders et al., 2023a; Linares-Mustarós et al., 2022). There is no other reason than agreement to use one ratio or its permutation. For a single firm, the fact that $x_1/x_2=0.5$ provides the same information as the fact that $x_2/x_1=2$. However, in statistical analyses, the results of the one and the other ratio may stand in contradiction. These ratio permutations are common in practice. The well-known solvency ratio or equity to assets ratio is the inversion of the leverage ratio in Equation (5):

$$\text{Solvency=equity/assets}=(x_4-x_3)/x_4. \tag{8}$$

Finally, if one of the accounting figures being compared in the ratio is close to zero, it may lead to an outlying standard ratio when placed in the denominator and to a typical ratio when placed in the numerator (Ezzamel and Mar-Molinero, 1990; Frecka and Hopwood, 1983; Kane et al., 1998; Lev and Sunder, 1979; Martikainen et al., 1995). For plr, placement makes no difference (Coenders et al., 2023a; Linares-Mustarós et al., 2018; 2022; Molas-Colomer et al., 2024).

Table 1 shows a toy example of seven fictional firms and two accounting figures x_1 and x_2. For ease of computation, we show logarithms to base 10 represented as $\log_{10}(x)$, which just tell how many times 10 has to be multiplied by itself in order to get the desired value.

$\log_{10}(1{,}000{,}000)=6$ because $10^6=1{,}000{,}000$. There is perfect symmetry around 1 and 0: $0.000001 = 1/1{,}000{,}000 = 1/10^6 = 10^{-6}$, so that $\log_{10}(0.000001) = -6$.

The interpretation of the plr with respect to standard ratios is straightforward. When x_1/x_2 is larger than 1, $\log_{10}(x_1/x_2)$ is positive. When x_1/x_2 is smaller than 1, $\log_{10}(x_1/x_2)$ is negative. The larger x_1/x_2, the larger $\log_{10}(x_1/x_2)$.

Note that, like ratios, logarithms focus on relative differences between firms. Ratios and logarithms are thus mutually compatible (Stevens, 1946) and should be routinely used together for data in a ratio scale, when the meaningful difference between two figures is relative, meaning that it lies in their ratio and not in their subtraction.

For example, if we take firms 3,4, and 5 in the toy example in Table 1 (x_2 values 100, 1,000 and 10,000), in relative terms, the difference between 1,000 and 100, which is 1,000/100=10, is the same as the relative difference between 10,000 and 1,000, which is 10,000/1,000=10. Accordingly, their log differences 3–2=1 and 4–3=1 are the same.

Note that the values of x_1 and x_2 in Table 1 are fully symmetrical while the standard ratios x_1/x_2 and x_2/x_1 are not symmetrical at all. In the ratio x_1/x_2, firms 1 and 2 appear as outliers and, in the ratio x_2/x_1, firms 6 and 7. In the ratio x_1/x_2, firms 4, 5, 6, and 7 are concentrated in the very short [0, 1] interval. In the ratio x_2/x_1 the same holds for firms 1, 2, 3, and 4.

Conversely, the logarithms of the ratios $\log_{10}(x_2/x_1)$ and $\log_{10}(x_1/x_2)$ are fully symmetrical, have no outliers, and permutation of numerator and denominator only leads to a sign reversal.

Table 1. Toy example with seven firms showing the symmetry of log-ratios and the asymmetry of standard ratios

Firm	x_1	x_2	x_2/x_1	x_1/x_2	$\log_{10}(x_1)$	$\log_{10}(x_2)$	$\log_{10}(x_2/x_1)$	$\log_{10}(x_1/x_2)$
1	1,000,000	1	0.000001	1,000,000	6	0	–6	6
2	100,000	10	0.0001	10,000	5	1	–4	4
3	10,000	100	0.01	100	4	2	–2	2
4	1,000	1,000	1	1	3	3	0	0
5	100	10,000	100	0.01	2	4	2	–2
6	10	100,000	10,000	0.0001	1	5	4	–4
7	1	1,000,000	1,000,000	0.000001	0	6	6	–6

Natural logarithms (to base e=2.718281828…) represented as $\log(x)$ are commoner in economics, finance, and CoDa but any base could be used without affecting the analysis.

Some plr between pairs of accounting figures are especially interesting in DuPont analysis. By definition, turnover compares revenues and assets:

$$y_1 = \log\left(\frac{x_1}{x_4}\right). \tag{9}$$

As shown in Equation (2), comparing revenues and costs provides a notion of margin:

$$y_2 = \log\left(\frac{x_1}{x_2}\right). \tag{10}$$

In the same vein, comparing liabilities and assets provides a notion of leverage. Even if it does not correspond with the standard leverage definition in Equation (5), higher liabilities with respect to assets do imply higher leverage:

$$y_3 = \log\left(\frac{x_3}{x_4}\right). \tag{11}$$

The inversion of leverage as a solvency indicator can indeed be performed, but it is just the original leverage indicator with a reversed sign $-y_3 = \log(x_4/x_3)$.

Potentially, $D(D\text{-}1)/2$ different plr can be computed, although some of them may fail to have any financial interpretation or theoretical interest, ratio choice becoming a potentially problematic issue. Great care must also be taken to prevent ratios from being mutually redundant, meaning that the information of some ratios is already contained in other (Barnes, 1987; Chen and Shimerda, 1981; Magrini, 2025; Pohlman and Hollinger, 1981). For instance, in the above example, through simple algebra it can be shown that a plr computed as the ability of revenues to pay for liabilities $y_4 = \log(x_1/x_3)$ is equal to $y_1 - y_3$:

$$\begin{aligned} y_1 - y_3 &= \log\left(\frac{x_1}{x_4}\right) - \log\left(\frac{x_3}{x_4}\right) = \log(x_1) - \log(x_4) - \left(\log(x_3) - \log(x_4)\right) = \\ &\log(x_1) - \log(x_3) = \log\left(\frac{x_1}{x_3}\right) = y_4. \end{aligned} \tag{12}$$

Greenacre (2019); and Creixans-Tenas et al. (2919) recommend drawing a graph in which the accounting figures are *vertices* (*nodes*), and the plr are *connections* (*edges*). In order to prevent redundancy, the graph must necessarily be connected and acyclic. This means that:

- It is possible to join any two accounting figures following the connections (i.e., the plr).
- There may not be closed circuits, that is, there is only one possible path to join any two accounting figures.

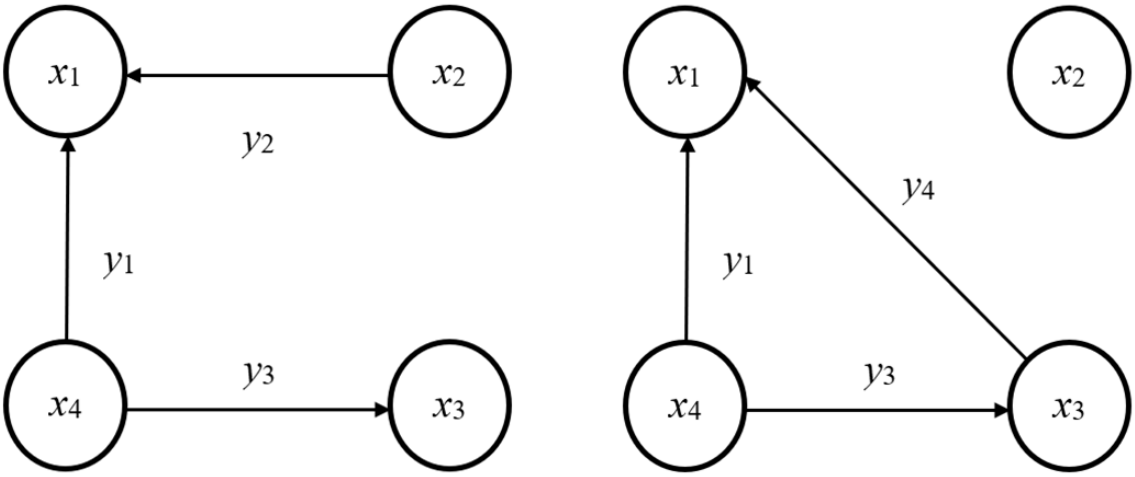
An acyclic connected graph has exactly $D-1$ plr (Greenacre, 2019). D-1 plr so chosen can be proven to contain all information about the D-part compositional dataset, in other words all information about the relative importance of the D accounting figures.

While any graph fulfilling these conditions will do the job, statistically speaking, it is good practice to use a graph based on expert knowledge on the interpretation of the ratios. In our DuPont analysis case, we want plr to be related to the concepts of turnover, margin and leverage, namely y_1, y_2 and y_3, which fortunately fulfil the conditions according to

the connected acyclic graph in the left panel of Figure 2. Edges can be drawn as arrows without affecting the graph properties, the arrows pointing at the numerator of the plr for clarification purposes only. In other words, accounting figures are considered joined even when going against the arrow directions.

As an example of inappropriate plr choice, when substituting $y_4 = \log(x_1/x_3)$ in Equation (12) for y_2, there would be a cycle connecting x_1, x_3 and x_4 (in other words, one could go from x_1 to x_3 either directly or through x_4) while x_2 would not be connected to any of the other parts (right panel in Figure 2).

Figure 2. Connected acyclic graph (left). Unconnected cyclic graph (right)



Users must be warned that there may be more than one way to choose a sensible set of D-1 interpretable and non-redundant plr, and the results of some statistical analyses which are based on distances (e.g., biplots and principal component analysis as used in Section 5) depend on this choice (Hron et al., 2021). These methods require alternative log-ratios, as shown in Section 3.2. Conversely, plr are appropriate for statistical inference (Section 6).

3.2. Centred log-ratios

Centred log-ratios -clr- (Aitchison, 1983) dispense with log-ratio choice and also contain all information about the relative importance of D accounting figures. clr compare each part, in the numerator, with the *geometric mean* of all parts for each individual firm, in the denominator:

$$
\begin{aligned}
clr_1 &= \log\left(\frac{x_1}{\sqrt[4]{x_1x_2x_3x_4}}\right) \\
clr_2 &= \log\left(\frac{x_2}{\sqrt[4]{x_1x_2x_3x_4}}\right) \\
clr_3 &= \log\left(\frac{x_3}{\sqrt[4]{x_1x_2x_3x_4}}\right) \\
clr_4 &= \log\left(\frac{x_4}{\sqrt[4]{x_1x_2x_3x_4}}\right).
\end{aligned}
\tag{13}
$$

All possible plr are contained in the clr. As an example, note how y_1 can be obtained from clr_1 and clr_4 by just subtracting them:

$$
\begin{aligned}
&clr_1 - clr_4 = \log\left(\frac{x_1}{\sqrt[4]{x_1x_2x_3x_4}}\right) - \log\left(\frac{x_4}{\sqrt[4]{x_1x_2x_3x_4}}\right) = \\
&\log(x_1) - \log\left(\sqrt[4]{x_1x_2x_3x_4}\right) - \left(\log(x_4) - \log\left(\sqrt[4]{x_1x_2x_3x_4}\right)\right) = \\
&\log(x_1) - \log(x_4) = \log\left(\frac{x_1}{x_4}\right) = y_1.
\end{aligned}
\tag{14}
$$

The clr have no accounting interpretation in themselves, but they are used as raw data in multivariate descriptive analysis methods such as principal component analysis and biplots, as shown in Section 5, while the interpretation is made in terms of plr.

3.3. Zero replacement

A commonly mentioned limitation of CoDa is that the accounting figures of interest may contain no zero values for log-ratios to be computed (Martín-Fernández et al., 2011). However, a fact that is often overlooked is that exactly the same holds for standard financial ratios: a zero accounting figure is not relative to anything and thus the ratio is not a feasible operation according to measurement theory (Stevens, 1946). The ratio is used to measure how many times a magnitude contains another, and this has no answer when one of the magnitudes is zero. If the zero value is the denominator, the standard ratio cannot even be computed.

Unlike the case in standard financial ratio analysis, CoDa include an advanced toolbox for *zero imputation* (a.k.a. *zero replacement*) prior to log-ratio computation under the most common assumptions (Martín Fernández et al., 2012). This provides CoDa with a head advantage compared to standard financial ratio analysis in the presence of zeros and ultimately makes financial statement analysis possible even when some accounting figures of interest equal zero. Shortly stated, zeros are replaced with a meaningful small value following certain statistical properties. See Mariadassou and Coenders (2025); and Coenders et al. (2023b) for recent reviews on the topic.

In the reviewed literature of compositional financial statement analysis, the most popular imputation method by far is the *log-ratio Expectation-Maximization* (*EM*) *method* (Palarea-Albaladejo and Martín-Fernández, 2008). This method is similar to the standard EM method for imputing missing data, as the imputed value is predicted from the available values with a statistical model. However, in the compositional case it adds the restriction that the imputed values must be "small". In particular, they are constrained to be below the minimum observed value of each part or below any other value specified by the user, called *detection limit*.

User-defined detection limits are particularly useful in the following case. If the minimum non-zero value corresponds to a firm with a very low figure, replacement below this limit could cause replaced values to be outliers. In this case, we recommend setting the detection limit to be a bit higher.

The zero imputation methodologies require the number of zero values to be small, ideally below 20 % for any of the accounting figures (Palarea-Albaladejo and Martín-Fernández, 2008). Before imputation, percentages of zeros should thus be examined. This may impede dividing assets and liabilities into very detailed accounts, such as buildings, trade names, inventory, accounts receivable, marketable securities, accounts payable, short-term loans, bonds, long-term loans, capital leases, and so on, some of which are zero for a large portion of firms, especially if the sample contains small and medium enterprises.

In other words, the choice of the number and detail of the D accounting figures has to be subject to the presence of zeros. If some accounting figures contain more than 20 % zeros, the user may want to sum them with other conceptually similar accounting figures with fewer zeros and thus reduce D. These aggregations are called amalgamations in the CoDa literature.

Related to the zero problem, inactive firms, as revealed by having zero revenues and/or zero assets should be removed from the dataset. If firms are inactive, they just do not belong to the study population, and it makes no sense to replace their missing accounting information with any sort of meaningful small value (just imagine what margin or turnover would look like with revenues or assets replaced with very small values). This situation is called *essential zeros*, *structural zeros*, *absolute zeros* or *true zeros* in the CoDa literature.

4. Example data

The financial statements in this example were obtained from the SABI (Iberian Balance sheet Analysis System, accessible at https://sabi.bvdinfo.com/) database, developed by *INFORMA D&B* in collaboration with *Bureau Van Dijk*. Search criteria were wine producers in Spain (NACE Rev.2 classification code 11.02 "manufacture of wine from grape") with available data for 2016. Inactive firms with zero revenues and/or zero assets were removed from the dataset. The final sample size after filtering inactive firms was n=109 and there were no remaining zero values. When using few and broad account categories as in DuPont analysis, zeros are rarely a problem (Saus-Sala et al., 2021; 2023).

In addition to x_1 to x_4, we consider the qualitative variable indicating if the firm sells at least some products using its own brand. This indicator is of especial interest, since it

reflects two winery business models which have deep implications. Firms without brands tend to sell non-bottled young wines at lower prices, while branded wines tend to be aged and expensive. Thus, firms without brands tend to have lower margins but higher turnovers and firms with brands the opposed characteristics. They constitute two *strategic groups* pursuing high return on equity through two different means. This dataset was also used in Linares-Mustarós et al. (2022); Coenders and Arimany-Serrat (2023); and Coenders (2025).

All analyses were carried out with CoDaPack2.03.10 (Comas-Cufí and Thió-Henestrosa, 2011; Thió-Henestrosa and Martín-Fernández, 2005), a menu-driven freeware available at https://ima.udg.edu/codapack/ .

The *boxplot* is an exploratory graphical display showing the division of firms into four equal-sized groups. Below the box there are 25 % of firms with the lowest values. Above the box there are 25 % of firms with the highest values. The line dividing the box is the median. Half of the firms have values below and above the median and also half of the firms have values within the box edges. Outliers are identified as separate points and the overall appearance of the boxplot tells about symmetry.

As reported in the reviewed literature, plr and clr (Figures 4 and 5) tend to be better behaved than standard ratios (Figure 3) in terms of asymmetry and outliers. In our example, the standard leverage ratio has one especially extreme outlier and strong asymmetry. The standard turnover ratio has strong asymmetry. Neither plr nor clr exhibit strong asymmetry or any extreme outlier. Moderate outliers are usually not harmful to the results of statistical analysis. Extreme outliers are.

Figure 3. Boxplots of standard ratios in Equations (3) to (5) showing extreme asymmetry and extreme outliers

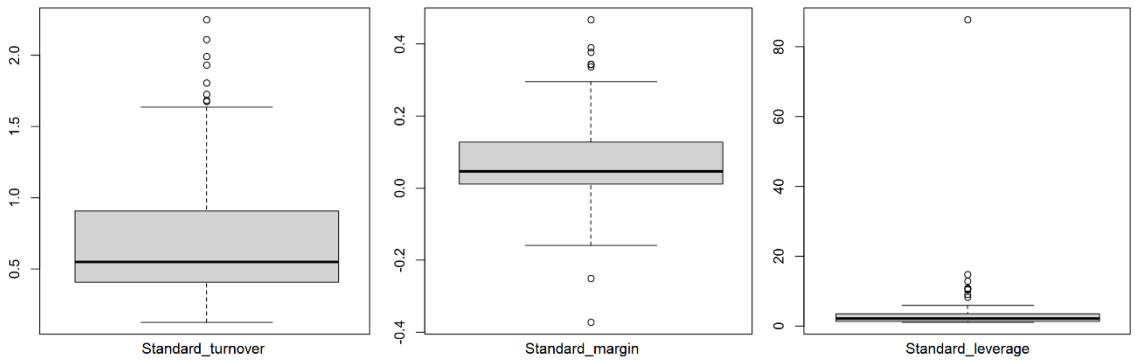


Figure 4. Boxplots of plr in Equations (9) to (11) showing at most mild asymmetry and few and non-extreme outliers

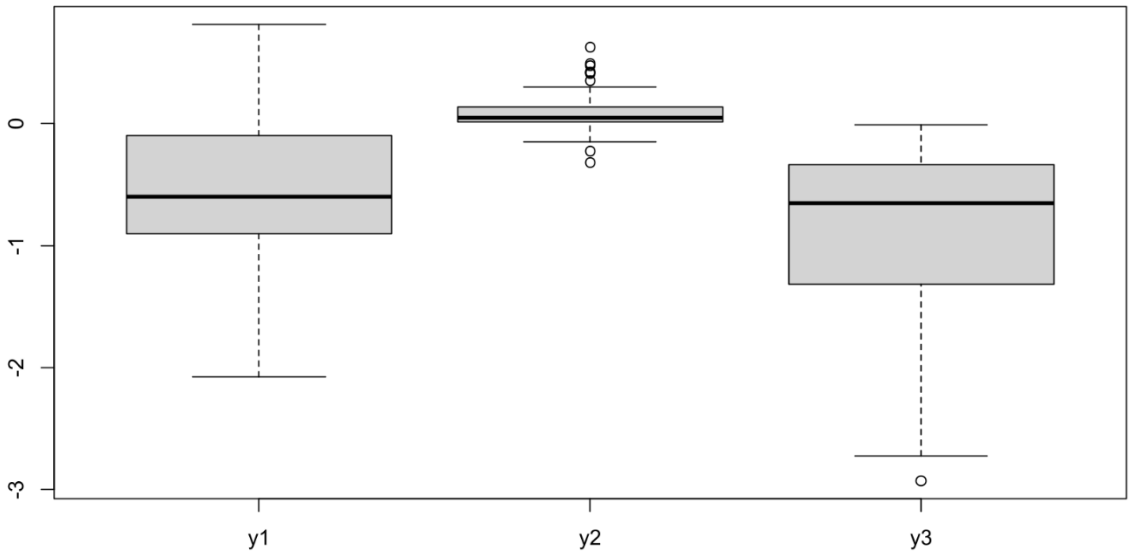
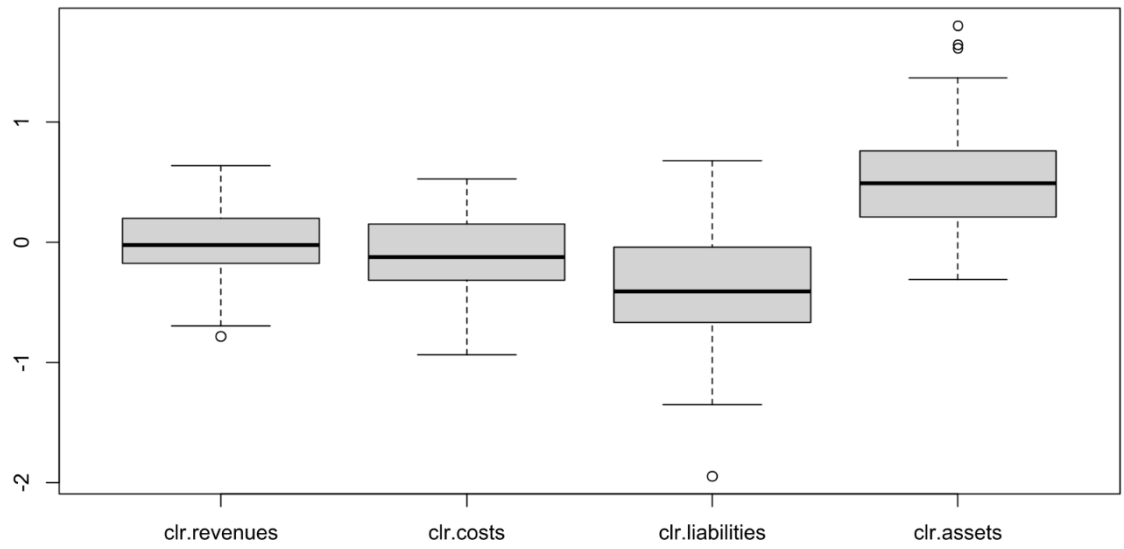


Figure 5. Boxplots of clr in Equation (13) labelled according to the accounting category in the numerator showing at most mild asymmetry and few and non-extreme outliers



5. Visualisation of ratios and individual firms with the CoDa biplot

Beyond simple graphics of one or two variables, multivariate visualization tools help researchers interpret large data tables with many firms and parts in just one graph. To this end, Aitchison (1983) extended the well-known *principal component analysis* procedure to the compositional case by using the *D* clr in Equation (13). A compositional principal component analysis computes a small number of uncorrelated dimensions, which explain the highest possible portion of the variances of all clr. In this way the original data set with potentially many clr can be summarized with just a few dimensions which are suitable for a graphical display.

The two first dimensions are represented in the so-called covariance *CoDa biplot* (Aitchison and Greenacre, 2002), which can be understood as the most accurate graphical representation of a compositional data set in two dimensions. The goodness of fit is indicated by the percentage of explained variance of the clr by the first two dimensions. In our example, the percentage of explained variance is very high at 99.0 % thus arguing for an extremely good biplot accuracy. Percentages above 70 % are considered acceptable, above 80 % good and above 90 % very good.

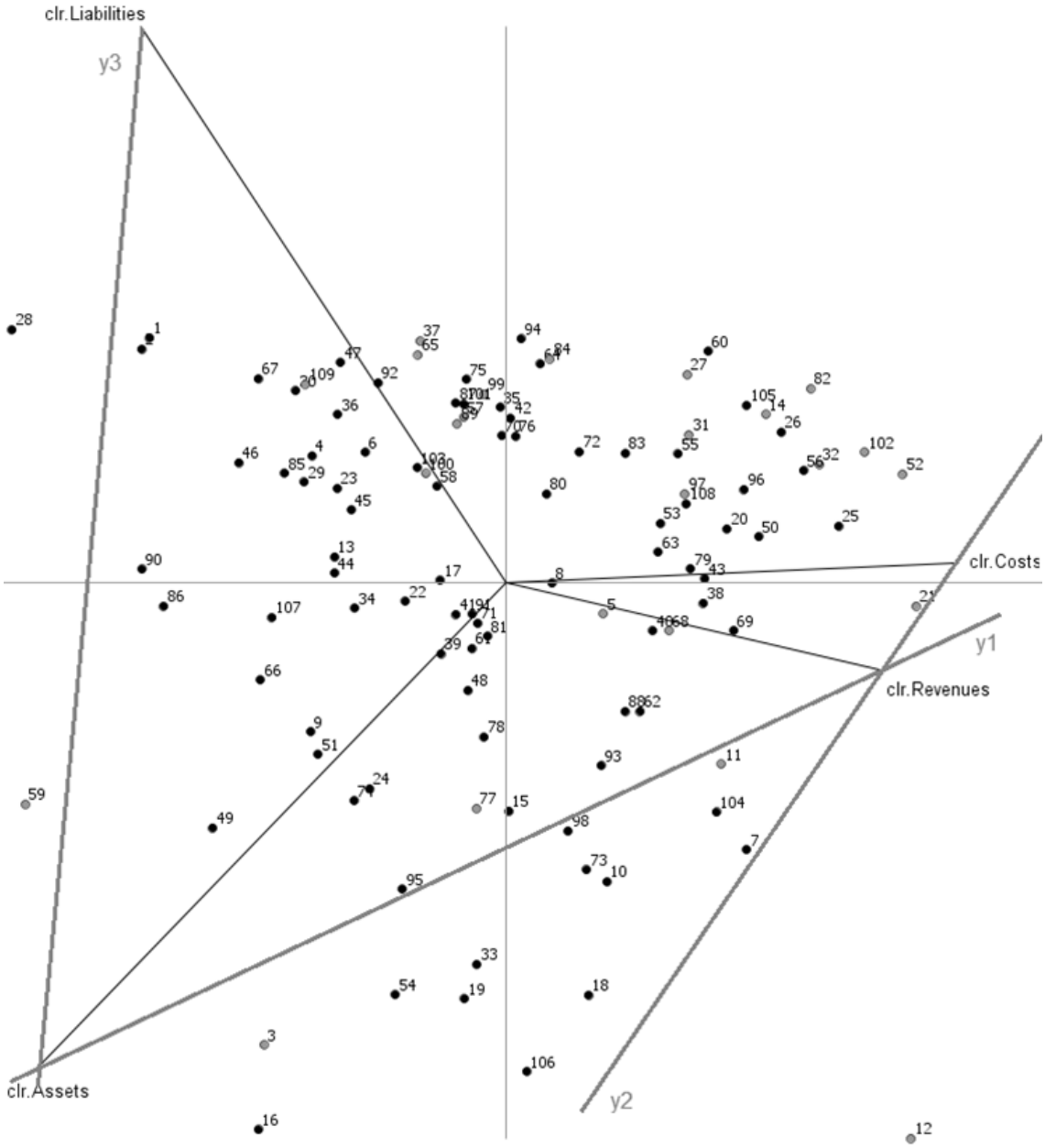
The CoDa biplot for financial statement data plots each clr representing the accounting figure in the numerator as a line called *ray*. Individual firms appear as points. Carreras-Simó and Coenders (2020); and Saus-Sala et al. (2021; 2023) highlight the most important interpretational tool of the CoDa biplot in financial statement analysis. Additional lines can be drawn linking the extremes of a pair of rays and representing the plr between the corresponding two accounting figures of the numerators of the clr. These additional lines are called *links*. The *orthogonal projection* of all firms along the direction defined by the link shows an approximate ordering of firms according to the plr between the corresponding two accounting figures. The orthogonal projection is made by dropping firms on the link in such a way that the direction in which the firms fall forms a 90-degree angle with the link.

In this way, the CoDa biplot is also a visual representation of any of the $D(D\text{-}1)/2$ possible financial ratios computed from any two accounting figures. The user can draw as many links as he or she wishes. Since the analysis is anyway run on clr, redundancy is not a problem, although only long links showing high variance plr tend to lead to informative directions. Thus, plr should not be drawn when the links are very short, in other words, when the vertices of the rays are close together.

The ability to visually interpret ratios between any two accounting figures is of great interest in financial statement analysis as it allows researchers to quickly identify the financial profile of individual firms (Carreras-Simó and Coenders, 2020; Coenders, 2025; Coenders and Arimany-Serrat, 2025; Rondós-Casas et al., 2026; Saus-Sala et al., 2021; 2023). The y_1 line representing turnover links the vertices of revenues and assets in Figure 6. Firms situated at the top and to the right (in the high-revenue side of the link) are the ones with the highest turnover. Firms situated at the bottom and to the left (in the high-asset side of the link) are the ones with the lowest turnover. The y_2 line representing margin links the vertices of revenues and costs. Firms situated at the bottom and to the left are the ones with the highest margin and firms situated at the top and to the right are the ones with the lowest margin. The y_3 line representing leverage links liabilities and assets. Firms at the top of the biplot are the most leveraged.

When firms are dropped forming a 90-degree angle on each of the y_1, y_2 and y_3 lines, firm 16 has the highest orthogonal projection on y_2 and the lowest on y_3. Thus, it is a firm with a very high margin and a very low leverage. Firm 59 has a very low turnover and firm 52 has a very high turnover. As a whole, firms without a brand (marked as grey) have a comparatively lower margin, higher leverage and higher turnover than firms with a brand (marked as black). The closest firms to the origin of coordinates (industry average) are 8, 17, 41, 71, 81 and 91.

Figure 6. CoDa biplot with added turnover (y1) margin (y2) and leverage (y3) directions. Firms with an own brand in black, without in grey. Firms can be projected on the y1, y2 and y3 directions



6. Statistical inference by groups of firms

Up to now we have dealt with descriptive statistical methods. This section is devoted to statistical inference and testing with the aim of drawing conclusions about the population from which the sample of firms was drawn. To this end, compositional financial ratios act as variables in any statistical model or test together with any firm characteristic. clr are not directly interpretable, while in statistical testing the interpretation of the included variables is a crucial issue, which makes plr a preferable option.

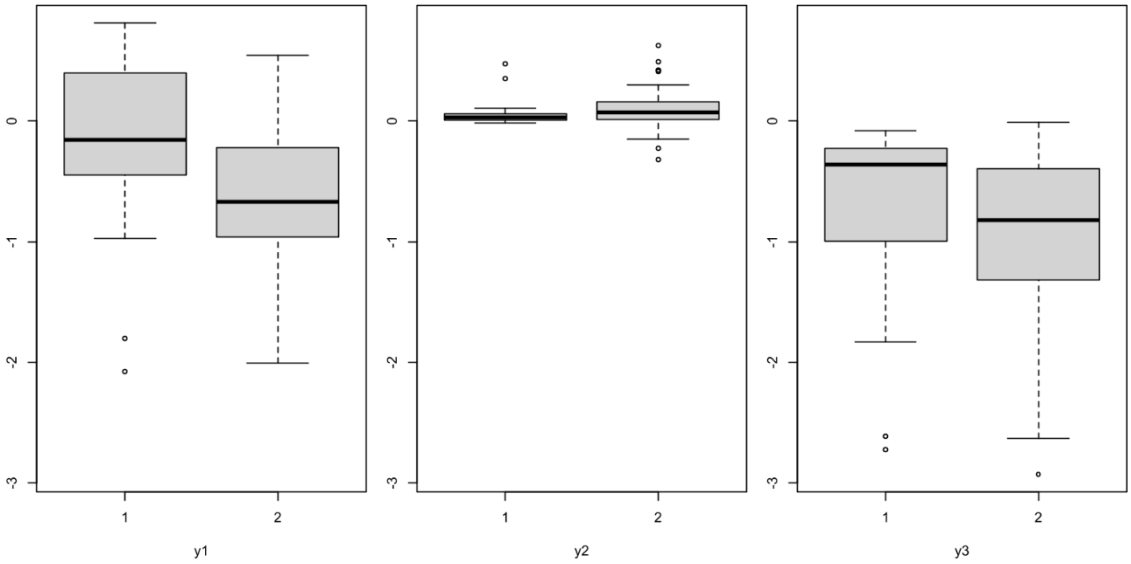
We consider the simplest possible case in which we want to test the equality of the plr between two groups of firms having or not an own brand (Linares-Mustarós et al., 2022;

Coenders et al., 2023a). Before testing, some graphical display relating the plr with the groups is very useful. According to the boxplots (Figure 7) firms with a brand have higher margin (y_2) but lower turnover (y_1) and lower leverage (y_3) in the sample. Testing will then refer to the population differences, if any.

The following statistical hypothesis is tested for each plr indicating that firms without (1) and with (2) a brand brand have the same population average plr (μ):

H_0: $\mu_1=\mu_2=0$

Figure 7. Boxplots of turnover (y1) margin (y2) and leverage (y3), for wineries without (1) and with (2) their own brand showing sample differences between groups



The *p-value* associated to each statistical hypothesis indicates the risk involved in rejecting the hypothesis. If it is low (e.g., lower than 0.05), the hypothesis can be rejected with the conclusion that the averages of the plr at hand differ significantly in the population. The test used is the well-known *t-test for two independent samples*:

$$t = \frac{\overline{x}_2 - \overline{x}_1}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \qquad (15)$$

where $\overline{x}$ s^2 and n stand for the sample mean, variance, and number of sample cases for firms without (1) and with (2) a brand.

Table 2. Independent sample t-test results

	$\overline{x}_2 - \overline{x}_1$	t	p-value
y_1 (turnover plr)	–0.409	–2.922	0.004
y_2 (margin plr)	0.037	1.178	0.241
y_3 (leverage plr)	–0.214	–1.335	0.185
$-y_3$ (permuted leverage plr)	0.214	1.335	0.185
Standard leverage ratio in Equation (5)	-0.791	-0.401	0.689
Permuted standard leverage ratio: solvency in Equation (8)	0.126	2.163	0.033

In the three first rows of Table 2, the p-values indicate that only turnover is significantly different for firms with and without brand (0.004 < 0.05). The negative sign of $\overline{x}_2 - \overline{x}_1$ indicates that firm with an own brand (2) have a lower turnover than firms without a brand. As expected, when permuting the numerator and denominator of y_3, the results are the same with a reversed sign. On the contrary, when doing the analysis on standard financial ratios, permutation does make a difference. When using the standard leverage ratio in Equation (5) no differences can be established between groups (p-value = 0.689 > 0.05), and when using the standard solvency ratio in Equation (8) firms with a brand have higher solvency, i.e., lower leverage (p-value = 0.033 < 0.05, positive sign of $\overline{x}_2 - \overline{x}_1$). In any case, these results on the standard ratios are suspect due to the extreme outliers in Figure 3. Linares-Mustarós et al. (2022); and Coenders et al. (2023a) obtained similar results.

7. Concluding remarks

7.1. Extensions and future perspectives in accounting research

Compositional Data (CoDa) can be used to advantage to distil the relative magnitude of accounting figures, which lies at the ground of researchers' and professionals' interests when computing financial ratios. CoDa solve the main drawbacks of standard financial ratios in statistical analysis, such as the extreme skewness and outliers shown in Figure 3, and the dependence of the results on the arbitrary permutation of numerator and denominator shown in Table 2. Over ten years, the accounting community has learnt that the violations of the assumptions of standard ratios can be severe, and that the improved CoDa methodology leads to results that are different enough to modify (for the better) the main conclusions of applied research. In this respect, the CoDa approach represents a vast improvement in applied accounting research using statistical methods on financial ratios. We can only encourage its widespread adoption, which should neither affect the researchers' objectives nor their statistical tastes.

Classical multivariate analysis tools can be used on CoDa after transforming the data by means of clr. In particular, the principal-component-analysis biplot visualizes all firms with respect to the ratio of any pair of accounting figures. The biplot can immediately be extended to panel data to plot trajectories of individual firms over time (Saus-Sala et al., 2023). Simple statistical inference is also possible by means of $D-1$ plr. plr can also play the role of dependent or explanatory variables in a linear regression model (Coenders and Arimany-Serrat, 2023; 2025; Farreras-Noguer et al., 2025; Mulet-Forteza et al., 2024)

and in other statistical models (Creixans-Tenas et al., 2019).

The reviewed applications in Section 1 show a variety of fields of study and research questions. The CoDa Methodology can potentially be applied to any research project which uses a statistical sample of firms and some of the variables of interest are financial ratios. The population of interest may be any industry or meaningful group of industries, or any other firm category of interest (by region, size, societal type, etc.).

Any positive non-overlapping accounting figures may be used to define any set of ratios, beyond the very simple DuPont-analysis case (Saus-Sala et al., 2021; 2023). For instance, current and non-current assets could have been used instead of total assets, and current and non-current liabilities could have been used instead of total liabilities bringing the number of accounting figures D to 6. This would have made it possible to define ratios of asset structure (non-current over current assets), debt maturity (non-current over current liabilities), and short-term solvency (current assets over current liabilities). See Arimany-Serrat and Coenders (2025); Arimany-Serrat et al. (2023); Coenders (2025); Creixans-Tenas et al. (2019); Dao et al. (2024); Hernandez-Romero and Coenders (2025); Jofre-Campuzano and Coenders (2022); and Saus-Sala et al. (2024). So far, the highest number of accounting figures found in the literature is D=14 (Carreras-Simó and Coenders, 2020).

To use accounting figures from other financial statements than the balance sheet and the income statement is also possible. Arimany-Serrat et al. (2022) extend the CoDa methods to the analysis of the cash-flow statement. It is also possible to include non-financial figures if they result into meaningful ratios. For instance, the number of employees appears in the average-wage ratio, the sales-per-employee ratio, the assets-per-employee ratio, and so on (Carreras-Simó and Coenders, 2020), and the number of rooms is a key variable to define management ratios in hotels (Mulet-Forteza et al., 2024). Another source of ratios is in the environmental, social and governance indicators. For instance, the ratio of energy consumption on revenues is defined as energy intensity and can be treated compositionally (Rondós-Casas et al., 2026; Todorov and Simonacci, 2020). Similar intensity ratios exist for water consumption, green-house gas emissions, waste generation, and so on.

CoDa is not needed when there is no substantial statistical analysis of a data sample. This includes case-study research, in which typically the evolution of key standard ratios is plotted over time for just one or two firms, and all qualitative research paradigms. It is not either needed to monitor the performance of one's own firm. However, CoDa is necessary to compute reliable means of the ratios at the industry level. So, comparing one's own firm to the industry average requires industry averages to be properly computed (Hernandez-Romero and Coenders, 2025; Saus-Sala et al., 2021). CoDa is not needed either when using statistical methods that are robust to extreme asymmetry and outliers, which incudes but is not limited to certain non-parametric methods (Iotti et al., 2023; 2024a; 2024b; 2024c).

There are of course interpretational trade-offs. Standard ratios have a simpler interpretation. This has indeed been a matter of concern. To this end, most cluster analysis applications back-transform compositional cluster means into standard ratios (Arimany-Serrat and Coenders, 2025; Arimany-Serrat and Sgorla, 2024; Coenders, 2025; Coenders and Arimany-Serrat, 2023; 2025; Dao et al., 2024; Hernandez-Romero and Coenders,

2025; Jofre-Campuzano and Coenders, 2022; Saus-Sala et al., 2021; 2023; 2024). The same is done when computing ratio averages at industry level (Arimany-Serrat and Coenders, 2025; Arimany-Serrat and Sgorla, 2024). In statistical modelling, the researchers' interests may lie in statistical testing, for which purpose the matter is irrelevant, or in the estimation of effect sizes, for which purpose it is very relevant. Farreras-Noguer et al. (2026) show how to interpret CoDa effect sizes in terms of standard ratios. A promising development is the computation of logarithms to base 2. Increasing the log-ratio by one unit is tantamount to doubling the standard ratio (Müller et al., 2018). Elasticities can also be used to advantage in the interpretation of log-ratio effects (Coenders et al., 2025).

7.2. Current methodological debates, potential application difficulties and challenges for further research

A current methodological debate in financial-ratio application is the choice of log-ratio transformation. Apart from plr and clr, another log-ratio transformation is also common in the applied works cited -ilr- (Egozcue et al., 2003; Egozcue and Pawlowsky-Glahn, 2005). There is not much to choose from in distance-based methods such as cluster analysis, but ilr are a powerful alternative to plr in statistical modelling. In most cases, rather than being a technical matter, the choice depends on the researcher's favourite way to interpret the results. Ilr have been used in Carreras-Simó and Coenders (2021); Escaramís and Arbussà (2025); and Magrini (2025). Coenders and Arimany-Serrat (2023) compare plr and ilr in practice. In this article we have chosen to present plr for their greater simplicity.

Another current debate is the role of weights. In financial applications of CoDa methods, they have so far only been assessed on cluster analysis (Dao et al., 2024; Jofre-Campuzano and Coenders, 2022) with substantial improvements on the interpretability of the results. They have potential for other distance-based methods such as biplots. This is a promising avenue for further research.

The main obstacle to the practical applicability of ratios (be they standard or compositional) is the presence of zeros. As stated in section 3.3, a too detailed subdivision into a large number D of accounting figures may not be advisable on the grounds of large percentages of zeros, especially if the sample contains small firms. From the experience in the reviewed literature, D=4 as used in the examples here is fine even when the sample contains micro enterprises. D=6, as defined above, is fine for small firms, and D>6 should be used for medium to large firms only.

Some statistical methods have been used only rarely in conjunction with CoDa, or not at all. This includes penalized regression (Saperas-Riera et al., 2024), Bayesian computation (Coenders et al, 2025), and many recent machine learning methods, which are becoming popular in financial forecasting problems with standard ratios (Martin-Melero et al, 2025). For some of these methods neither plr, clr or ilr may be the best transformation, which is also a promising avenue for further research.

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AUTHOR CONTRIBUTIONS

Conceptualization: NAS and GC; methodology: GC; formal analysis: GC; data curation: NAS; writing - original draft: GC; writing - review & editing: NAS; Supervision: NAS; funding acquisition: GC.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY

The data are available in Coenders and Arimany-Serrat (2023).

USE OF ARTIFICIAL INTELLIGENCE (AI)

AI tools were not used in any part of the article.

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